Regulating Capital Flows to Emerging Markets: An Externality View

Anton Korinek*
Johns Hopkins University

December 2013

Abstract

This paper provides welfare theoretic foundations for risk-adjusted capital flow regulations based on a standard class of macroeconomic models of financial crises that exhibit financial amplification dynamics. We show that during crisis episodes when such amplification effects are triggered, decentralized agents do not internalize that capital outflows are magnified through a systemic feedback cycle of depreciating exchange rates, tightening financial constraints, and declining aggregate demand, akin to Fisher's process of debt deflation. As a result, agents undervalue the social cost of repayments in crisis states and take on too much systemic crisis risk in their ex ante financing decisions. We construct an externality kernel that captures the state-contingent magnitude of systemic externalities of outflows. Constrained social efficiency can be restored by imposing Pigovian taxes on capital inflows that can be calculated as the product of the externality kernel times the state-contingent vector of payoffs of the respective type of financial instrument. We develop a sufficient statistics method to quantify the externalities imposed by different categories of capital flows. Using historical data from Indonesia, we find that optimal Pigovian taxes range from approximately zero for FDI flows to 1.5% for foreign currency-denominated debt.

JEL Codes: F41, E44, D62, H23
Keywords: financial crises, financial amplification, capital flow regulation, externality kernel

*The author would like to thank Viral Acharya, Patrick Bolton, Alessandra Bonfiglioli, Phil Brock, Fernando Broner, Tiago Cavalcanti, Mick Devereux, Walt Fisher, Gita Gopinath, Olivier Jeanne, Alessandro Rebucci, Carmen Reinhart, Joseph Stiglitz, Cédric Tille, Carlos Vegh and Iván Werning as well as participants at several conferences and seminars for helpful discussions and comments. For contact information see `http://www.korinek.com/`

1 Introduction

In the aftermath of the 2008/09 global financial crisis, emerging market economies around the world have experienced a resumption of strong capital inflows, as their growth prospects appeared superior to those of the industrialized world. This has led to renewed concern among academics and policymakers that the strong boom in capital inflows would create financial fragilities in emerging economies that could lead to a painful bust when global appetite for investment in such economies reversed. As a consequence, a number of countries, including Brazil and Taiwan, have recently enacted restrictions on capital inflows.[1] With the tapering of quantitative easing in the US, capital flows have reversed their direction, validating these fears and leading to concerns about financial stabilities in countries such as Turkey, Indonesia, Brazil, Russia and India.

In standard neoclassical models (see e.g. Obstfeld and Rogoff, 1996) there is no role for restrictions on capital flows, since free international capital markets allow poor countries to increase their capital stock and insure against idiosyncratic shocks, thereby raising growth and reducing consumption volatility. However, Gourinchas and Jeanne (2006) point out that in a calibrated version of a neoclassical model, the welfare benefits of financial integration are actually quite small. Furthermore, empirical evidence such as Reinhart and Reinhart (2008) suggests that capital inflows to emerging market economies often create financial vulnerabilities that both increase consumption volatility and hurt growth prospects.

A number of academics, including Bhagwati (1998), Rodrik (1998) and Stiglitz (2002) have used such observations to argue against financial market liberalization. They pointed out that the welfare theorems that underlie the case for capital market liberalization only hold in economies that suffer from no other distortions. In emerging market economies that are rife of market imperfections, restrictions on capital accounts may be an optimal policy in a second-best sense. However, this literature did not provide a detailed economic mechanism that would explain why the decentralized equilibrium in such an economy would be inefficient, and what exact forms of regulation would be warranted.

This paper sets out to provide welfare theoretic foundations to this question. We analyze the constrained efficiency of the financing and risk-taking decisions of private agents in emerging market economies that are prone to financial crises. We describe financial crises as situations in which financial amplification dynamics arise and a feedback cycle of depreciating exchange rates, tightening financial constraints, and declining aggregate demand is triggered, akin to Fisher's process of debt deflation.[2]

[1] See e.g. "Capital inflows create dilemma," *Financial Times, US edition*, Apr. 21, 2010, p. 4.

[2] Such models of financial amplification have become the leading tools to describe the dynamics of financial crises from both a conceptual and a quantitative point of view. Other macroeconomic models of crises that fall into this category include for example Fisher (1933); Kiyotaki and Moore (1997); Bernanke et al. (1999); Gertler and Kiyotaki (2010) or, in the emerging market context, Krugman (1999); Christiano et al. (2004); Jeanne and Zettelmayer (2005); Mendoza (2005); Devereux and Yetman (2009); Céspedes, Chang and Velasco (2012). All these papers share the common element that borrowers are subject to financial constraints that endogenously depend on prices, and when constraints bind adverse feedback effects arise. In the literature, other common terms for what we

Our main finding is that rational private agents do not optimally solve the trade-off between the benefits of foreign capital and the risks of financial crises because of a pecuniary externality: Decentralized agents take prices (including the exchange rate) as given. In perfect financial markets, this behavior leads to a Pareto efficient outcome. However, in an economy in which financial amplification effects are at work, agents are subject to financial constraints and the tightness of constraints endogenously depends on prices.

A decentralized agent who commits to repayments in constrained states of nature does not internalize that larger repayments in such states lead to larger capital outflows, which entail stronger exchange rate depreciations and tighter constraints. Therefore he undervalues the social cost of liabilities that mandate repayments in constrained states of nature. In other words, since he takes exchange rates as given, he does not internalize his contribution to the financial amplification dynamics. As a result of this externality, the welfare theorems no longer hold.

A constrained planner internalizes the link between the economy's aggregate financing decisions, the level of the exchange rate, and the tightness of financial constraints. When choosing the optimal composition of foreign liabilities, she recognizes that reducing repayments in states when financial amplification effects are active has the indirect effect of mitigating price declines and relaxing binding constraints on agents across the economy. Therefore the financing decisions of a constrained planner involve less risk-taking and less exposure to binding constraints than those of decentralized agents. The macroeconomic outcome is less volatility and less financial instability.

We show how well-targeted regulations on risky forms of capital inflows may alleviate this distortion and induce decentralized agents to switch towards safer forms of finance, thereby implementing the constrained social optimum.

In our analytical model, we study an emerging economy that is subject to endowment risk and financial constraints that endogenously depend on the value of collateralizable income. In a variety of settings, we analyze the constrained efficiency of the ex-ante financing and risk-sharing allocations of decentralized domestic agents that trade financial claims with international investors in a complete set of Arrow securities.

- First, we assume that international investors are risk-neutral to illustrate the effects of the externality in a simplified framework. Decentralized agents fully insure against consumption fluctuations in the period of the shock and contract a constant level of consumption across all states, including those in which financing constraints are binding and in which they would like to carry more debt into future periods. By contrast, a constrained planner carries extra "macro-prudential insurance" into constrained states, which relaxes the constraint and allows for better intertemporal consumption smoothing. In other words, the planner deviates from perfect intratemporal smoothing as a second-best device to improve intertemporal consumption smoothing.

- Next we investigate a framework where international investors are risk-averse to-

call "financial amplification" are debt deflation, financial acceleration, financial feedback loops, or deleveraging cycles.

wards the endowment shock in the domestic economy. The decentralized liability structure of domestic agents is determined by a risk-return trade-off and may involve exposure to binding constraints in low states of nature. Whenever constraints bind, a planner who internalizes the financial amplification effects would reduce the exposure of domestic agents to the constraint so as to mitigate financial amplification effects and relax the constraints.

- In our third setting, we study an emerging economy without endowment risk interacting with international investors that face certain states of the world towards which they are risk-averse. It is privately optimal for domestic agents to share risk with international investors. In particular, in states of high international risk aversion, domestic agents provide large payments to international investors, to the point that they may become constrained – they experience "contagion" from international capital markets. Again, a social planner who internalizes financial amplification effects would reduce exposure to such states so as to mitigate the amplification and relax financial constraints.

All three settings illustrate that full international risk-sharing is not desirable when financial markets are subject to collateral constraints that may trigger financial crises in the form of financial amplification. In normal times, emerging market economies may *seem* to be integrated into global markets, but their access to international financial markets is in fact state-contingent and is sharply reduced or lost whenever financing constraints on the economy become binding. Decentralized agents find it privately optimal to participate in global risk-sharing by taking on risky forms of finance, but they fail to internalize that the level of financial integration of their economy is endogenous: their private risk-taking decisions affect the tightness of constraints in states of crisis. A social planner engages in international risk sharing according to a tradeoff between risk, return, *and* the endogenous level of financial integration, as captured by the tightness of financial constraints. She therefore reduces the exposure of the economy to binding constraints.

Our paper constructs a *social pricing kernel* that prices emerging market liabilities at their true social cost. (In analogy to traditional pricing kernels, which reflect how much private agents value payoffs across different states of nature, the social pricing kernel is a random variable that expresses how much a social planner values payoffs across different states of nature.)

The difference between the private and social valuation of payoffs represents how much decentralized agents undervalue the social costs of state-contingent payoffs. We denote this difference an *externality kernel*. In unconstrained states of nature, private and social pricing kernels coincide and the externality kernel is zero. In constrained states, the social planner internalizes that payoffs are socially more costly than decentralized agents realize; the externality kernel is positive and grows larger the tighter financial constraints are.

Careful empirical analysis of different forms of capital flows reveals that the precise type of capital flows matters: Foreign currency-denominated debts seem to significantly magnify macroeconomic volatility and raise the risk of financial crisis without yielding benefits in terms of higher growth (Calvo et al., 2004; Levy Yeyati, 2006, see). On the other hand, financial flows that are conducive to risk-sharing, such as foreign direct investment, seem to be positively associated with both macroeconomic stability and long-run growth (see e.g. Mauro et al., 2007).

This corresponds closely to our analytical results: the magnitude of externalities and therefore of optimal policy measures depends crucially on the risk profile of the liabilities involved. For a given liability, the expected size of the externality can be calculated as the product of its stochastic vector of payoffs with the externality kernel.

This formula can be used by policymakers to calculate the social costs imposed by capital flows of different forms, such as dollar debt, GDP-linked debt, local currency debt, portfolio investment, or foreign direct investment. For example, foreign currency-denominated debt, which mandates high payoffs in crisis states, is associated with large externalities; by contrast, foreign direct investment, which typically pays no dividends in the midst of crises, involves close to zero externalities.[3]

Policy measures against the discussed externality are only effective if they affect marginal incentives for risk-taking. We derive a *bailout neutrality* result for bailouts in the form of lump-sum transfers: private agents who have access to a complete set of Arrow securities and who expect transfer payments in times of crisis – from their own government or from international bodies – will increase their risk-taking so as to fully undo expected transfers that are in expectation wealth-neutral, since the decentralized equilibrium with excessive risk taking constitutes their private optimum. This can be interpreted as a state-contingent version of Ricardian equivalence (see Barro, 1974).

We develop a novel method to quantify the magnitude of the externalities of financial amplification effects based on the sufficient statistics approach proposed by Chetty (2009). This allows us to quantify the externalities of financial amplification independently of the precise setup of the theoretical model chosen in the paper, and therefore to provide more robust numbers than what could be obtained from calibrating a particular version of a DSGE model of financial amplification, in which a comparatively large number of structural parameters have to be calibrated. We show that the externalities of financial amplification effects can be captured by the product of two sufficient statistics, (i) the wedge in the Euler equation of constrained agents and (ii) the extent of amplification, as captured by the elasticity of the tightness of the constraint to the underlying shock. Our approach therefore provides for a more direct, transparent and credible identification of the described externalities.

We apply this approach to Indonesia and illustrate how the externalities of different forms of capital flows can be quantified using data over the past 20 years. We find that during the 1997/98 crisis, the externalities associated with dollar debt, local currency debt and equity portfolio inflows were 30.7%, 8.9% and 6.2% respectively of the value of the inflow, implying an optimal annual level of taxation of those inflows of 1.54%, 0.44% and 0.31% respectively.

[3]These results contrast with Tobin (1978), since our policy measures would apply only to risky forms of finance. In addition, our measures are motivated from a well-specified externality rather than a general concern about the volatility of international capital flows.

Relationship to the Literature

Our work is related to a significant body of research on why emerging economies are often so strongly exposed to the risk of systemic financial crises. Two of the most common explanations in this literature involve moral hazard: First, private agents have incentive to take on excessive risk if they expect bailouts from their government or from international organizations.[4] Secondly, rigid forms of finance that impose considerable risk on borrowers (such as uncontingent debt, foreign currency debt, short-term debt) may be optimal incentive devices that prevent borrowers or their governments from engaging in actions that expropriate creditors, such as devaluation, a weakening of property rights, etc.[5] Our work offers an alternative and complementary approach to this literature: we show that socially excessive risk taking may result from a financial friction that exists even in the absence of governmental distortions, i.e. because of the externalities that arise from financial amplification dynamics.

Our work is most closely related to Caballero and Krishnamurthy (2003), who analyze excessive dollar borrowing in an economy that is subject to two financial frictions, in international and domestic credit markets.[6] Our approach differs in two important respects: First, our externality result derives from a single market friction in international capital markets and holds even though (i) domestic financial markets are frictionless and (ii) ex ante international capital markets are complete. Secondly, we use a simple macroeconomic model that allows us to quantify the magnitude of externalities of different type of capital flows.

A number of other recent papers have focused on reasons why countries may want to impose capital controls. For example, Farhi and Werning (2012, 2013) focus on how capital controls can correct for aggregate demand externalities that may be triggered under price rigidities. Costinot et al. (2011) investigate how capital can be used to exert monopoly power over a country's intertemporal terms-of-trade, i.e. to distort world interest rates. These are distinct types of externalities from the one we investigate.

Our work adds to the literature in two more dimensions: First, we conceptually introduce the framework of an externality kernel. This enables us to perform a detailed welfare analysis of emerging market capital flows and to provide clear theoretical foundations for regulations that discriminate among various forms of capital flows according to their risk profile. Secondly, we develop a sufficient statistics approach that allows us to calibrate the discussed externalities in a robust manner and independently of the precise model of financial amplification effects employed. We believe that this makes our conceptual framework a well-suited theoretical basis for policy evaluation, as we discuss in more detail in sections 6 and 7 of the paper.

In methodology, our work contributes to the literature on financial amplification

[4]See e.g. Krugman (1998); Schneider and Tornell (2004); Rancière (2009) for examples of this literature. However, Eichengreen and Hausmann (1999) note that risky forms of finance are pervasive even among firms that are unlikely to be bailed out, and in most emerging market crises government bailouts are not sufficient to cover most of the firms that go bankrupt.

[5]See Jeanne (2003); Tirole (2003); Jeanne (2009) for example.

[6]For a related analysis in DSGE frameworks with debt only see e.g. Bianchi (2011); Benigno et al. (2013). For a related analysis of overborrowing in the closed economy based on fire-sale externalities in asset prices see Lorenzoni (2008).

effects, which has often been invoked as one the main mechanisms to describe financial crises (see footnote 2 but we focus on the normative implications of such mechanisms, in particular on the associated pecuniary externalities.

The remainder of the paper is structured as follows. Section 2 introduces our benchmark model of a small open emerging market economy in which financial amplification is triggered in low states. Sections 3 and 4 contrast the decentralized equilibrium with the allocation chosen by a constrained planner. Section 5 illustrates the basic inefficiency results in a number of analytical examples. Section 6 analyzes policy measures to correct the distortion. Finally, section 7 quantifies the externalities and the ensuing optimal policy measures for the case of Indonesia using a sufficient statistics approach.

2 Model Setup

We consider a small open emerging market economy in infinite discrete time $t = 0, 1, 2, \ldots$. The economy consists of a continuum of mass 1 of identical domestic agents. There are two goods in the economy, a tradable good T which is the numeraire good, and a non-tradable good N with a relative price p_N. This price represents a measure of the real exchange rate.[7] Starting in period 1, domestic agents derive utility from the consumption of tradable C_T and non-tradable goods C_N according to the utility function

$$U = E\left\{\sum_{t=1}^{\infty} \beta^t u(C_t^{1+\sigma})\right\} \quad \text{where} \quad C_t = C_{T,t}^{\frac{1}{1+\sigma}} C_{N,t}^{\frac{\sigma}{1+\sigma}} \tag{1}$$

where the consumption index C_t is a Cobb-Douglas aggregator of tradable and non-tradable goods with shares $\frac{1}{1+\sigma}$ and $\frac{\sigma}{1+\sigma}$, implying that the ratio of the value of non-tradable to tradable consumption is σ.[8] The utility function $u(\cdot)$ is a standard twice continuously differentiable function satisfying $u'' < 0 < u'$. The discount factor of domestic agents is given by β.

In period 0, domestic agents are born with initial debt B_0. They have to meet a fixed investment requirement $\bar{I}$ every period, and in return they earn a bundle of tradable and non-tradable goods $(Y_{T,t}^{\omega}, Y_{N,t}^{\omega})$ every period starting from period 1, which depends on the aggregate state of the economy $\omega \in \Omega$.[9]

Domestic agents have access to a full set of one-period Arrow securities that pay out contingent on the state $\omega \in \Omega$ of the world. We denote the payoff that domestic agents promise in state ω of period $t+1$ as B_{t+1}^{ω}. International investors pay $E[M_{t+1}^{\omega} B_{t+1}^{\omega}]$ in period t for a bundle $\left\{B_{t+1}^{\omega}\right\}$ of such promises in period $t+1$, where M_{t+1}^{ω} represents the

[7] Our model of the exchange rate captures the real exchange rate and accounts for the stylized fact that exchange rates depreciate in crises, i.e. in response to strong negative shocks. For brevity, we will often refer to the real exchange rate simply as the "exchange rate" in the remainder of the paper.

[8] The exponent $1+\sigma$ in the utility function is included for notational convenience in our later derivations. Our qualitative results are unaffected by this transformation − the notation can accommodate any desired utility function $\hat{u}(C)$ that is sufficiently concave by defining $u(C) = \hat{u}\left(C^{\frac{1}{1+\sigma}}\right)$.

[9] The investment requirement serves to motivate borrowing, but does not play a critical role in our model. It could be replaced by a commensurately higher initial debt level.

pricing kernel (stochastic discount factor) of international investors and is exogenous to the small open economy. Investors hence require a return of $R^\omega_{t+1} = 1/M^\omega_{t+1}$ on a security contingent on state ω.

Assumption 1 *We assume the pricing kernel of international investors satisfies* $E[M^\omega_{t+1}] = \beta \ \forall \ t$ *and is monotonically decreasing in domestic endowment risk,*

$$\forall \omega_1, \omega_2 \in \Omega : Y^{\omega_1}_{T,t} < Y^{\omega_2}_{T,t} \Rightarrow M^{\omega_1}_t \geq M^{\omega_2}_t$$

The first part of the assumption implies that domestic agents and international investors are equally patient so that the risk-free rate $R = 1/E[M^\omega_{t+1}]$ offered by international investors satisfies $\beta R = 1$. The second part implies that international investors are (weakly) risk-averse towards tradable endowment shocks in the emerging economy. We will discuss the implications of relaxing this assumption in section 5.3.

The budget constraints of domestic agents for period 0 and beyond are

$$\bar{I} + B_0 = E[M^\omega_1 B^\omega_1] \tag{2}$$

$$C^\omega_{T,t} + p^\omega_{N,t} C^\omega_{N,t} + \bar{I} + B^\omega_t = E[M^\omega_{t+1} B^\omega_{t+1}] + Y^\omega_{T,t} + p^\omega_{N,t} Y^\omega_{N,t} \quad \text{for } t \geq 1, \omega \in \Omega \tag{3}$$

We make two simplifying assumptions regarding the structure of endowment risk:[10] First, we assume all realizations of the tradable endowment for $t \geq 2$ are constant at a level $\bar{Y}_T$, which coincides with the expected period 1 endowment $E[Y^\omega_{T,1}] = \bar{Y}_T$. Secondly, we assume the non-tradable endowment is fixed at a level $\bar{Y}_N$, which we normalize w.l.o.g. to $\bar{Y}_N = 1$.[11] These assumptions allow us to treat the state of the world ω as an ordinal variable that follows the order $\omega_1 \leq \omega_2$ whenever $Y^{\omega_1}_{T,1} \leq Y^{\omega_2}_{T,1}$.

While financial markets in the economy are complete in the sense that a contingent Arrow security for each state exists, we assume that emerging market agents are subject to a moral hazard problem: when they receive finance, they have an option to invest into fraudulent schemes that would allow them to default next period (they could e.g. invest into assets that are different from what was agreed upon and that are harder to seize, or they could set up a shell company that funnels the money out of their financiers' reach). International investors can detect this and fight the scheme in court while the transaction is in progress, but because of imperfect legal enforcement they can recover at most a fraction κ of the domestic agent's income in that period. Furthermore, coordination problems among international investors prevent them from taking retaliatory actions to penalize default through exclusion in future periods.

In order to make it incentive-compatible for domestic agents to abstain from fraud, investors limit the value of outstanding claims $E[M^\omega_{t+1} B^\omega_{t+1}]$ to the maximum that they could recover in case of fraud:

$$E[M^\omega_{t+1} B^\omega_{t+1}] \leq \kappa \big(Y^\omega_{T,t} + p^\omega_{N,t} \bar{Y}_{N,t} \big) \tag{4}$$

[10] These assumptions allow us to obtain closed form solutions for most expressions below, which is useful to analytically characterize the constrained social welfare properties of the economy.

[11] This simplifying assumption allows us to explicitly solve for the level of the real exchange rate. Our finding that the real exchange rate depreciates during crises would still hold if the shock affected the endowments of both sectors $(Y^\omega_{T,t}, Y^\omega_{N,t})$ symmetrically, since crises in our setup are accompanied by current account reversals that magnify the effects of shocks on tradable consumption.

This incentive compatibility constraint implies that the financing capacity of domestic agents is limited by the amount of their seizable assets (or "collateral"), which is an increasing function of their tradable income and of the economy's real exchange rate.[12] If the real exchange rate depreciates, the domestic borrowing capacity declines, which captures the common notion in the literature that exchange rate depreciations in emerging economies may be contractionary (see e.g. Krugman, 1999; Aghion et al., 2004; Christiano et al., 2004) In the remainder of the paper, we will identify periods when financial constraints on the representative domestic agent are binding so that financial amplification effects are active as financial crises.

We make the following assumption to rule out degenerate equilibria:

Assumption 2 *We assume that $\kappa\sigma < 1$.*

This guarantees that amplification effects are finite, i.e. that the exchange rate appreciation resulting from a one dollar capital inflow raises the financing capacity of the country by less than one dollar.

3 Decentralized Equilibrium

Domestic agents maximize the objective function (1) subject to the series of budget constraints (2) and (3), to which we assign the shadow prices of liquidity μ_t^ω, as well as the financial constraints (4) with shadow prices λ_t^ω. The Lagrangian of this problem is given in expression (A.1) in the appendix. The first-order conditions are

$$\mathrm{FOC}(C_{N,t}^\omega): \quad \sigma u'(\cdot) C_{T,t} C_{N,t}^{\sigma-1} = \mu_t^\omega p_{N,t}^\omega \tag{5}$$

$$\mathrm{FOC}(C_{T,t}^\omega): \quad \mu_t^\omega = u'(\cdot) C_{N,t}^\sigma \tag{6}$$

$$\mathrm{FOC}(B_{t+1}^\omega): \quad \mu_t^\omega M_{t+1}^\omega = \beta \mu_{t+1}^\omega + \lambda_t^\omega M_{t+1}^\omega \tag{7}$$

The decentralized equilibrium in the described economy consists of

- a bundle of allocations $(C_{T,t}^\omega, C_{N,t}^\omega, B_{t+1}^\omega)$ and prices $p_{N,t}^\omega$ for $t = 1, 2, ...$ and $\omega \in \Omega$
- which solve the constrained optimization problem of decentralized agents and
- which clear markets in all periods and states
  - for non-tradable goods: $C_{N,t}^\omega = \bar{Y}_N$
  - for tradable goods: $C_{T,t}^\omega + \bar{I} + B_t^\omega = Y_{T,t}^\omega + E(M_{t+1}^\omega B_{t+1}^\omega)$

[12] It is a common practice that international lenders provide finance to domestic agents against both tradable and non-tradable collateral. For example, many real estate booms are fuelled by foreign credit based on non-tradable collateral. Foreign creditors who seize non-tradable assets from a defaulting borrower can sell it in the domestic market and repatriate the receipts.

Real Exchange Rate

Combining the first-order conditions on $C_{T,t}$ and $C_{N,t}$ pins down the relative price of non-tradables in a given time period, i.e. the real exchange rate of the economy, as

$$p_{N,t}^{\omega} = MRS = \sigma \cdot \frac{C_{T,t}^{\omega}}{C_{N,t}^{\omega}} = \sigma C_{T,t}^{\omega} \tag{8}$$

where we used the market-clearing condition for the non-tradable sector $C_{N,t}^{\omega} = Y_{N,t}^{\omega} = 1$ in the last step. Intuitively, a shock to the domestic consumer's tradable wealth affects his demand for both tradable and non-tradable goods as both goods are normal. Since the consumption of non-tradable goods is fixed to $\bar{Y}_N$, the relative price in non-tradables adjusts in response to changes in demand for non-tradables.

The movement of $p_{N,t}^{\omega}$ constitutes a pecuniary externality. It is the mechanism by which equilibrium in the market for non-tradable goods is restored. Under normal circumstances, pecuniary externalities have no adverse efficiency implications as they constitute mere redistributions between the buyers and sellers of a good. However, in the described economy the valuation of domestic collateral depends on the level of the exchange rate. When financial constraints are binding, a depreciation in the exchange rate reduces the value of collateral, which reduces agents' financing capacity and forces them to cut back on borrowing. As we show below, when financing constraints are binding, the pecuniary externality turns into a 'real' externality.

Solution by Backward Induction

We first solve for the equilibrium in the economy from period 1 onwards, taking the endowment shock $Y_{T,1}^{\omega}$ and the period 1 obligation B_1^{ω} as given; after that we will investigate the optimal risk allocation in period 0. Dropping the ω-superscripts, let us denote the utility of a domestic consumer who enters period 1 with a given endowment shock $Y_{T,1}$ and a financial obligation B_1 as

$$\begin{aligned} V(Y_{T,1}, B_1) = \max \sum_{t=1}^{\infty} \beta^t u\left(C_{T,t}\right) \\ \text{s.t.} \quad B_t + \bar{I} + C_{T,t} = Y_{T,t} + B_{t+1}/R \\ B_{t+1}/R \leq \kappa\big(Y_{T,t} + p_{N,t}\bar{Y}_N\big) \end{aligned} \tag{9}$$

where $M_t = 1/R \; \forall t \geq 2$ since there is no further uncertainty after period 1, and where we substitute the market-clearing condition $C_{N,t} = \bar{Y}_N = 1$. Combining and simplifying the first-order conditions (6) and (7), the Euler equation of the domestic agent from period 1 onwards is

$$u'(C_{T,t}) = \beta R u'(C_{T,t+1}) + \lambda_t \tag{10}$$

If the financial constraint on the agent is always loose ($\lambda_t = 0 \; \forall \; t$), then he absorbs the period 1 uncertainty (i.e. the deviation of $Y_{T,1}$ from $\bar{Y}_T$) into his steady-state debt level $B_{t\geq 2} = B_1 - (Y_{T,1} - \bar{Y}_T)$ and chooses a smooth steady-state consumption path along which he consumes the constant amount

$$\bar{C}_T(B_{t\geq 2}) = \bar{Y}_T - \bar{I} - \frac{R-1}{R} \cdot B_{t\geq 2} \tag{11}$$

where we denote variables that have reached their steady-state levels starting in period s by a subindex $t \geq s$. Every period, the agent consumes his net endowment minus the interest accumulated on his steady state debt $B_{t\geq 2}$, which is carried forward into the next period. The associated period 1 levels of consumption and new borrowing are

$$C^{unc}_{T,1}(Y_{T,1}, B_1) = \bar{C}_T\left(B_{t\geq 2}\right) = \bar{Y}_T - \bar{I} - \frac{R-1}{R}\left(B_1 + \bar{Y}_T - Y_{T,1}\right) \tag{12}$$

$$B^{unc}_2(Y_{T,1}, B_1) = B_{t\geq 2} = B_1 - (Y_{T,1} - \bar{Y}_T) \tag{13}$$

The financial constraint in period 1 is binding if

$$B^{unc}_2(Y_{T,1}, B_1)/R > \kappa\left[Y_{T,1} + \sigma C^{unc}_{T,1}(Y_{T,1}, B_1)\right]$$

This constraint defines a threshold level $\hat{B}(Y_{T,1})$ that is increasing in the output shock $Y_{T,1}$, such that the constraint binds if $B_2 > \hat{B}(Y_{T,1})$ (see appendix A.3). Under binding constraints the period 1 levels of consumption and new borrowing are determined by the level of the constraint

$$C^{con}_{T,1} = Y_{T,1} - \bar{I} - B_1 + B^{con}_2/R$$
$$\text{and}\quad B^{con}_2/R = \kappa\left[Y_{T,1} + \sigma C^{con}_{T,1}\right]$$

Solving this system of two equations explicitly yields

$$C^{con}_{T,1}(Y_{T,1}, B_1) = \frac{(1+\kappa)Y_{T,1} - \bar{I} - B_1}{1-\kappa\sigma} \tag{14}$$

$$B^{con}_2(Y_{T,1}, B_1) = \frac{\kappa R}{1-\kappa\sigma}\cdot\left[(1+\sigma)Y_{T,1} - \sigma\left(\bar{I} + B_1\right)\right] \tag{15}$$

From period 2 onwards, the tradable endowment is constant at $\bar{Y}_T$. If $Y_{T,1} \leq \bar{Y}_T$, then the constraint relaxes between periods 1 and 2, and the B^{con}_2 that satisfies the period 1 financial constraint also satisfies all future financial constraints, implying that the economy's long-run levels of debt and consumption are $B_{t\geq 2} = B^{\text{con}}_2$ and $C_{T,t\geq 2} = \bar{C}_T(B^{\text{con}}_2)$ $\forall t \geq 1$. On the other hand, if $Y_{T,1} > \bar{Y}_T$, the constraint would tighten from period 1 to period 2 and may become binding in period 2.[13] Since binding constraints and financial crises do not ususally occur in periods of above-average output $Y_{T,1} > \bar{Y}_T$, we restrict our attention to initial debt levels that are sufficiently low to rule out this case:

Assumption 3 *The economy's initial debt level is sufficiently low $B_0 \leq \bar{B}_0$ so that financial constraints only bind in below average states of nature $Y^{\omega}_{T,1} \leq \bar{Y}_T$.*

We analytically characterize the threshold level $\bar{B}_0$ in appendix (A.4). For higher initial debt levels, the agent carries so much debt into period 1 that constraints also bind in some states with above average realizations of output.

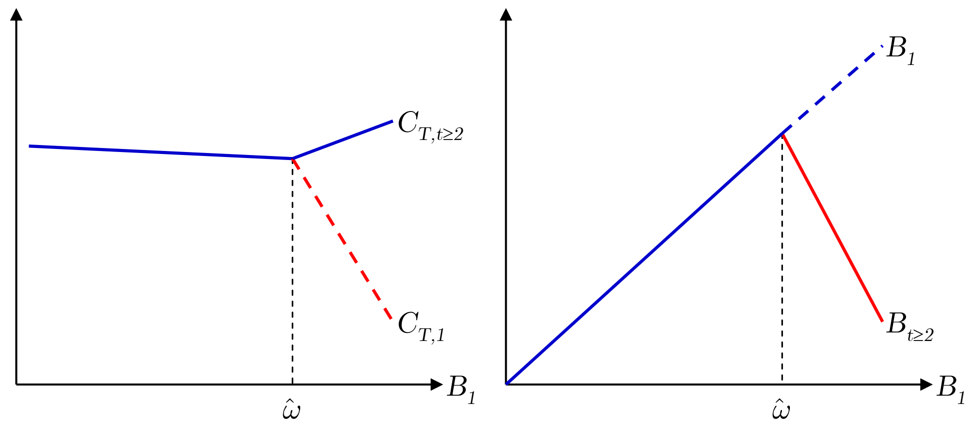

Figure 1: Consumption (left panel) and desired/realized future debt (right panel) as a function of initial obligation B_1 for a given state ω

Comparative Statics of Period 1 Debt

Figure 1 plots consumption in periods $t = 1$ and $t > 1$ (left panel) as well as steady-state debt (right panel) as a function of the amount of debt B_1^ω for a given output shock $Y_{T,1}$. To the left of $\hat{B}_1$, financing constraints are loose and agents can smooth their debt repayment over time. Consumption reacts only mildly to changes in initial debt (left panel) as the debt burden is carried over into the future (right panel).

To the right of $\hat{B}_1$, the financial constraint is binding and any change in the domestic agent's net worth is amplified through financial amplification. Assume for instance that financial constraints are binding and we increase the agent's debt burden B_1 by one dollar. If his financing capacity were unaffected, he would have to contract his consumption $C_{T,1}^\omega$ by one dollar. However, the reduction in consumption depreciates the exchange rate (8) by σ, and this depreciation in turn tightens the financing constraint by $\kappa\sigma$. The tighter constraint forces the agent to cut consumption by $\kappa\sigma$ dollars, which leads to further rounds of depreciating exchange rates, tightening constraints, declines in consumption and so forth. The total effect on consumption of a unit shock to the agent's budget constraint is $1 + \kappa\sigma + (\kappa\sigma)^2 + \cdots = \frac{1}{1-\kappa\sigma}$.

These contrasting effect of shocks to net worth on consumption in the constrained and unconstrained region are reflected in the derivatives

$$\frac{dC_{T,1}^{unc}}{dB_1} = -\frac{R-1}{R} \gg -1 \tag{16}$$

$$\frac{dC_{T,1}^{con}}{dB_1} = -\frac{1}{1-\kappa\sigma} < -1 \tag{17}$$

[13]The analysis of this case proceeds along the lines of equations (14) and (15) with the time index shifted forward by one period. We obtain the same externality result as for binding constraints in period 1. Detailed derivations of this case are available from the author.

Period 0 Risk Allocation Problem

Having solved for the equilibrium in the economy in periods 1 and after, we now focus on the financing and risk-taking decisions of decentralized agents in period 0 across different states of nature. Building on sub-problem (9) we denote the full optimization problem of decentralized agents as

$$\max_{\{(B_1^\omega)\}} E\{V(B_1^\omega, Y_{T,1}^\omega)\} \quad \text{s.t.} \quad \bar{I} + B_0 = E[M_1^\omega B_1^\omega] \tag{18}$$

Assigning a Lagrange multiplier μ_0 to the constraint, this leads to the first-order condition

$$V_B(B_1^\omega, Y_{T,1}^\omega) + \mu_0 M_1^\omega = 0 \qquad \text{or} \qquad \mu_1^\omega = \mu_0 M_1^\omega \tag{19}$$

where we applied the envelope condition $V_B = -\mu_1^\omega$. Substituting for the agent's valuation of liquidity μ_1^ω we obtain

$$u'(C_{T,1}^\omega) = \mu_0 M_1^\omega \tag{20}$$

Given the strict concavity of the utility function, this uniquely defines a function

$$C_{T,1}(M_1^\omega, \mu_0) = u'^{-1}(\mu_0 M_1^\omega) \tag{21}$$

which satisfies $\partial C_{T,1}/\partial M_1^\omega < 0$ and $\partial C_{T,1}/\partial \mu_0 < 0$: consumption in state ω is lower the higher the valuation of payoffs of international investors and the tighter the period 0 budget constraint, i.e. the higher the initial debt of the domestic agent.

This consumption function in conjunction with the budget constraint/borrowing constraint of domestic agents define a function $B_1(M_1^\omega, \mu_0)$ that captures the period 1 repayment consistent with the optimal level of consumption (see appendix A.2).

Lemma 1 *$B_1(M_1^\omega, \mu_0)$ is continuous in M_1^ω and μ_0 and satisfies $\partial B_1/\partial \mu_0 > 0$ and $\partial B_1/\partial M_1^\omega > 0$.*

Substituting into the period 0 budget constraint we obtain

$$E[M_1^\omega B_1(M_1^\omega, \mu_0)] = B_0 + \bar{I} \tag{22}$$

The left-hand side of this equation is strictly increasing in μ_0 since each B_1^ω is strictly increasing; the right-hand side is determined by the exogenous parameter B_0. Equation (22) therefore has a unique solution for the shadow price as a function of initial debt $\mu_0(B_0)$ with $\mu_0'(B_0) > 0$. This allows us to define a function $B_1(M_1^\omega, B_0) = B_1(M_1^\omega, \mu_0(B_0))$ that is strictly increasing in both arguments.

We define the threshold $\hat{B}_0$ as the maximum period 0 debt level such that the borrowing constraint is loose across all states of nature (see appendix A.2 for formal definition). Then the decentralized equilibrium in the economy as a function of the initial debt level B_0 is characterized by the following proposition:

Proposition 1 (Decentralized Equilibrium) *The optimal repayment function $B_1^\omega(M_1^\omega, B_0)$ is an increasing function of initial debt B_0 and of the pricing kernel M_1^ω. For $B_0 \le \hat{B}_0$,* *financial constraints are loose across all periods and states of the world. For $B_0 \in (\hat{B}_0, \bar{B}_0]$, the period 1 financial constraint is binding in all states ω below a threshold $\hat{\omega} = \hat{\omega}(B_0)$ and loose otherwise. In the constrained region, higher initial debt B_0 has two effects: First, it raises the threshold $\hat{\omega}$ and thereby increases the number of states in which constraints are binding in period 1. Secondly, it raises the promised level of period 1 repayments, which exacerbates the tightness of constraints in the constrained region.*

4 Constrained Social Planner's Equilibrium

This section contrasts the decentralized equilibrium with the allocations a constrained social planner would choose. We assume that the planner maximizes the same objective (1) and is subject to the same budget constraints (2), (3) and financial constraint (4) as decentralized agents. The only difference is that a social planner internalizes that her consumption allocations affect the real exchange rate, which in equilibrium determines the tightness of the financial constraint (4) through the condition $p^{\omega}_{N,t} = \sigma C^{\omega}_{T,t}$. We report the Lagrangian of the resulting optimization problem in appendix A.5.

The first-order conditions (5) and (7) on $C^{\omega}_{N,t}$ and B^{ω}_{t+1} are unaffected in the planner's equilibrium, but the optimality condition for $C^{\omega}_{T,t}$ changes to

$$\text{FOC}(C^{\omega}_{T,t}): \quad \mu^{\omega}_t = u'(C^{\omega}_{T,t}) + \kappa\sigma\lambda^{\omega}_t \tag{23}$$

Solution by Backward Induction

As in our characterization of the decentralized equilibrium, we solve the planner's optimization problem in two steps using backward induction. We first define the social planner's value function $V^{SP}(\cdot)$ as in problem (9) as the optimum utility value that can be obtained given a period 1 endowment $Y^{\omega}_{T,1}$ and repayment obligation B^{ω}_1. Then we will go back to period 0 and solve the maximization problem analogous to (18) using the planner's valuation function $V^{SP}(\cdot)$.

Under assumption 3 the financial constraint is binding at most in period 1 and $\lambda^{\omega}_t = 0$ in all other time periods. If the constraint is loose in period 1, then the decentralized and the planner's first-order conditions (6) and (23) are identical and their allocations coincide. If the constraint is binding, both decentralized agents and the planner borrow and consume the maximum amount dictated by the constraint, so their allocations also coincide. In short, in the given setup, the planner cannot improve upon the decentralized equilibrium once the state-contingent financing choices $\{(B^{\omega}_1)\}$ have been made.

However, by combining the two first-order conditions (23) and (7) we find

$$\lambda^{\omega,SP}_1 = \frac{u'(C^{\omega}_{T,1}) - u'(C^{\omega}_{T,2})}{1-\kappa\sigma} \tag{24}$$

$$\mu^{\omega,SP}_1 = u'(C^{\omega}_{T,1}) + \kappa\sigma\lambda^{\omega,SP}_1 = \frac{u'(C^{\omega}_{T,1}) - \kappa\sigma u'(C^{\omega}_{T,2})}{1-\kappa\sigma} \tag{25}$$

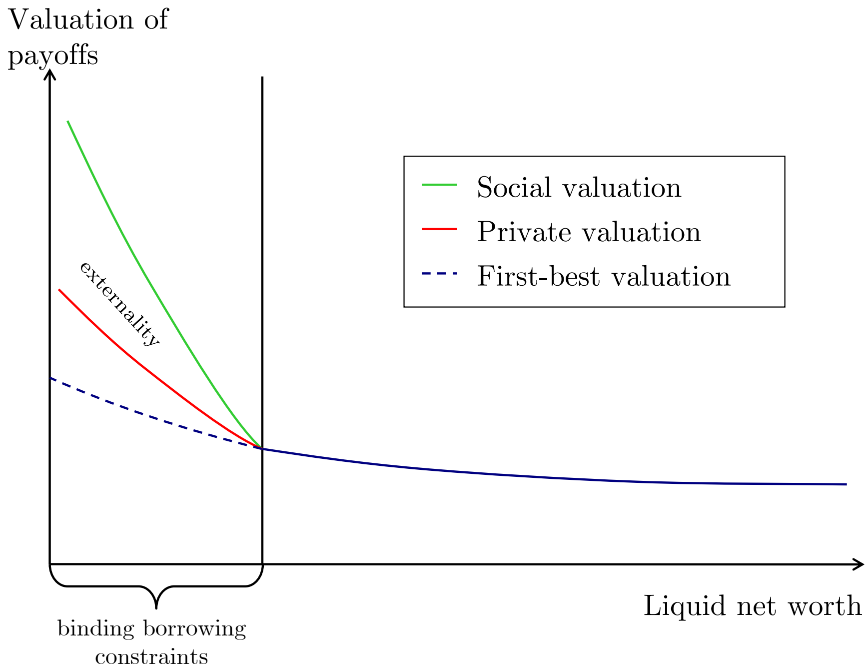

Figure 2: Private and social valuation of period 1 liquidity

Even though the planner's real allocations for a given pair $(Y_{T,1}^{\omega}, B_1^{\omega})$ coincide with those of decentralized agents, the shadow value that the planner assigns to period 1 consumption and to the binding constraint for a given real allocation differ from the decentralized equilibrium. For the planner, the shadow value of period 1 wealth $\mu_1^{\omega,SP}$ in constrained states is higher than for decentralized agents, because the planner internalizes the financial amplification effects: an exogenous unit increase in period 1 liquidity would raise consumption not only by one unit, but would also appreciate the exchange rate, increase the value of domestic collateral and relax the financial constraint further, ultimately leading to an increase in consumption by a factor $\frac{1}{1-\kappa\sigma}$.

Proposition 2 (Undervaluation of Liquidity in Crises) *In crisis states (when financial constraints are binding), the social planner values liquidity more highly than decentralized agents $\mu^{\omega,SP} > \mu^{\omega,DE}$ for a given real allocation, since she internalizes the financial amplification effects arising from binding constraints.*

Figure 2 schematically illustrates the shadow value of liquidity for different levels of liquid net worth (captured by varying B_1) for a given output shock $Y_{T,1}$. In a world without constraints (to the right of the vertical threshold line), changes in the debt level have relatively mild effects on the agent's valuation of liquidity, since he can spread repayments over the infinite future so as to smooth consumption. By contrast, when financial constraints are binding (to the left of the threshold, middle line), a decentralized agent's valuation of liquidity is higher than in the first-best world: the agent would like to borrow, but the constraint prevents him from doing so – his expected future wealth is not liquid and cannot be accessed because of the moral hazard problem. A planner (top line) internalizes that an increase in liquidity in a constrained state would raise consumption not merely one-for-one, but would also relax constraints and lead to positive financial amplification effects. Therefore the constrained planner's valuation of liquidity is even higher than that of constrained decentralized agents.

Period 0 Risk Allocation Problem

In period 0, the planner solves the same maximization problem (18) as decentralized agents and obtains the same first-order condition (19). However, the planner's shadow value of liquidity $\mu_1^{\omega,SP}$ reflects that her valuation of period 1 liquidity differs in constrained states, yielding the optimality condition

$$u'(C_{T,1}^{\omega}) + \kappa\sigma\lambda_1^{\omega} = \frac{u'(C_{T,1}^{\omega}) - \kappa\sigma u'(C_{T,2}^{\omega})}{1-\kappa\sigma} = \mu_0 M_1^{\omega} \tag{26}$$

In unconstrained states ω s.t. $\lambda_1^{\omega} = 0$, decentralized agents and the planner choose identical levels of repayments and consumption as determined by equations (A.2) and (21) for a given μ_0. By the same argument, the threshold $\hat{B}_0$ for initial debt to yield an equilibrium that is unconstrained across all states of nature is identical to that of the decentralized equilibrium:

Proposition 3 (Unconstrained Planner's Equilibrium) *For initial debt $B_0 \leq \hat{B}_0$, the social planner's allocation coincides with that of decentralized agents and financial constraints are always loose.*

In constrained states of nature, decentralized agents and the social planner allocate their repayments $\{(B_1^{\omega})\}$ differently across different states of nature:

Proposition 4 (Excessive Repayments in Constrained States) *A constrained planner chooses smaller repayments and higher consumption than decentralized agents in any constrained state $\omega \in \Omega^{con}(\mu_0)$ for a given μ_0,*

$$C_{T,1}^{con,SP}(M_1^{\omega}, \mu_0) > C_{T,1}(M_1^{\omega}, \mu_0) \qquad \textit{and} \qquad B_1^{con,SP}(M_1^{\omega}, \mu_0) < B_1^{con,DE}(M_1^{\omega}, \mu_0)$$

This can be seen by comparing the planner's optimality condition (26) with the decentralized version (20): when the constraint is binding, the additional term λ_1^{ω} introduces a positive wedge in the planner's equation. For given μ_0 and M_1^{ω}, this implies that the planner chooses a lower marginal product $u'(C_{T,1}^{\omega})$, i.e. a higher level of period 1 consumption, to make the equation hold.

In other words, the planner chooses smaller repayments or smaller exposure to binding constraints – she holds *macro-precautionary* savings or insurance against such states. This allows her to expand consumption in such states, which appreciates the exchange rate and indirectly relaxes the constraints. The planner distorts the optimal risk allocation in period 0 as a second-best device to mitigate the financial constraint in period 1.

We substitute the planner's financing choices into equation (22) and follow the same steps as in the solution to obtain the planner's solution for $\mu_0^{SP}(B_0)$. For a given μ_0, the planner repays less in constrained states $B_1^{con,SP} < B_1^{DE}$, reflecting the *macro-precautionary* savings effect. To raise the same total amount of finance in period 0, the planner's solution therefore requires a higher $\mu_0^{SP}(B_0) > \mu_0^{DE}(B_0)$ than the decentralized equilibrium. This raises the repayment across all states of nature since $\partial B_1^{SP}/\partial\mu_0 > 0$, in particular in unconstrained states. Furthermore, the higher μ_0 also raises the threshold below which financial constraints are binding $\hat{\omega}(\mu_0^{SP}) > \hat{\omega}(\mu_0^{DE})$, and thereby increases the probability of binding constraints. We sum up:[14]

Proposition 5 (Constrained Planner's Equilibrium) *For $B_0 \in (\hat{B}_0, \bar{B}_0^{SP}]$, the constrained planner's allocation involves a higher shadow price $\mu_0^{SP}(B_0) > \mu_0^{DE}(B_0)$ of period 0 liquidity than the decentralized allocation, resulting in a higher threshold $\hat{\omega}^{SP} = \hat{\omega}(\mu_0^{SP}(B_0))$ below which the period 1 financial constraint is binding.*

The net effect of reducing repayments in constrained states and then increasing repayments across the board so as to raise the same total amount of finance is to reallocate repayments from states of nature with strongly binding constraints to states with non-binding or marginally binding constraints.

The effects of changes in initial debt B_0 in this region are similar to those in the decentralized equilibrium (proposition 1): Higher initial debt raises the promised level of period 1 repayments B_1^ω in all states of nature. This in turn raises the threshold $\hat{\omega}^{SP}$ and increases the number of states in which constraints are binding in period 1. It also increases the tightness of the binding constraints and the size of the associated externalities.

4.1 Private and Social Pricing Kernel

So far we have analyzed period 0 financing and insurance decisions using a complete set of Arrow securities $\{(B_1^\omega)\}$. Here we extend our analysis to more complex securities, which can be thought of as bundles of Arrow securities with appropriate weights: any security X has a uniquely defined payoff in each state of nature ω of every period t:

Definition 1 *A security X is defined by a vector of state-contingent payoffs $\{(X_t^\omega)\}$.*

Since we assumed there is no further uncertainty after period 1, all securities are essentially risk-free bonds after period 1 and we may limit our focus on the risk allocation problem in period 0 of one period securities with payoffs X_1^ω. Note that this assumption is not restrictive: Since constraint (4) is imposed every period and securities can be bought and sold every period, any multi-period security with future payoffs $\{(X_t^\omega)_{t\geq 2}\}$ can alternatively be represented as a one-period security together with bond holdings of $B_2 = \sum_{t=2}^{\infty}(X_t/R^{t-1})$ in our framework. In other words, since the moral hazard problem in section 2 leads to the financial constraint (4) being imposed on the total value of all future repayment obligations every period, agents cannot circumvent the constraint by issuing long-term securities – they would still have an incentive to default if the current value of their long-term obligations exceeds the limit imposed by the constraint.

[14] The threshold $\bar{B}_0^{SP}$ is defined in appendix A.4.

We define the private pricing kernel of domestic agents as

$$D_1^\omega := \frac{\mu_1^\omega}{\mu_0} = \frac{\beta u'(C_{T,1}^\omega)}{E[u'(C_{T,1}^\omega)]}$$

This variable reflects the period 0 valuation of a unit payoff in state ω of period 1. Since there is no period 0 consumption in our setup, the marginal cost of tightening the period 0 budget constraint for domestic agents is $\mu_0 = RE[u'(C_{T,1}^\omega)]$ according to equation (19).

In period 0, international investors supply finance to domestic agents at a price determined by their pricing kernel M_1^ω. In the unregulated decentralized equilibrium, domestic agents adjust their portfolio so that their pricing kernel D_1^ω equals the pricing kernel M_1^ω of international investors. In the absence of imperfections, the period 0 valuation $P_{X,0}$ of an asset with payoffs X_1^ω in period 1 is therefore identical for decentralized domestic agents and international investors,

$$E[D_1^\omega X_1^\omega] = P_{X,0} = E[M_1^\omega X_1^\omega]$$

This does not account for the social costs that repayments in constrained states and the resulting exchange rate depreciations impose on the owners of domestic collateral. We define the social pricing kernel S_1^ω to reflect the social cost of a unit payoff in state ω of period 1 as

$$S_1^\omega := \frac{\mu_1^{\omega,SP}}{\mu_0^{SP}} = \frac{\beta u'(C_{T,1}^\omega) + \kappa\sigma\lambda_1^\omega}{E[u'(C_{T,1}^\omega) + \kappa\sigma\lambda_1^\omega]}$$

The social valuation $P_{X,0}^*$ of an asset X with payoffs X_1^ω is

$$P_{X,0}^* = E[S_1^\omega X_1^\omega] \tag{27}$$

Accounting for the fact that capital outflows create negative externalities in constrained states, the social price of risky assets is higher than the decentralized price $P_{X,0}$ demanded by international investors.

Externality Kernel

The greater the payoffs of a given security in crisis states, the larger the externality that the security imposes on the economy. Let us assume the economy is in the planner's equilibrium. We define the difference between the social and the private valuation of liquidity in that equilibrium, normalized by the expected private valuation, as the *externality kernel* τ_1^ω:

$$\tau_1^\omega = \frac{\mu_1^{\omega,SP} - \mu_1^{\omega,DE}}{E[\mu_1^{\omega,DE}]} = \kappa\sigma \cdot \frac{\lambda_1^{\omega,SP}}{E[\mu_1^{\omega,DE}]} = \frac{\kappa\sigma}{1-\kappa\sigma} \cdot \frac{u'(C_{T,1}^\omega) - \beta R u'(C_{T,2}^\omega)}{E[u'(C_{T,1}^\omega)]} \tag{28}$$

where we employed equation (24) in the last step. The intuition of this expression is the following: the uninternalized social benefit of a unit payoff in state ω is a relaxation of the financial constraint by $\kappa\sigma$, which is magnified by the factor $\frac{1}{1-\kappa\sigma}$ through financial amplification. The marginal increase in utility from relaxing the constraint is in turn $\left[u'(C^{\omega}_{T,1}) - \beta R u'(C^{\omega}_{T,2})\right]$, which is normalized by the expected marginal private value of liquidity $E[u'(C^{\omega}_{T,1})]$. The externality kernel is proportional to the wedge between the private and social valuation of payoffs in figure 2. It is zero when in states in which the financing constraint is loose. In states in which financing constraints are binding, a state-contingent payoff X^{ω}_1 entails an externality of size $E\left[\tau^{\omega}_1 X^{\omega}_1\right]$. We will employ this finding below in section 6 on optimal policy measures against the externality that we identified.

5 Analytical Examples

In this section, we illustrate the decentralized equilibrium allocation and the constrained social optimum of a sample economy under three scenarios. First we assume that international capital markets are risk-neutral, allowing for costless insurance of period 1 consumption. This helps to establish the basic intuition of our results. Secondly, we assume risk-averse international capital markets in which domestic agents determine their liability structure as the optimal trade-off between private risk and return. In a third scenario, we assume that the emerging market economy does not face any endowment risk, but international capital markets are risk-averse and induce domestic agents to share some of the global risk factors with them. In all three settings, we compare the allocation of the decentralized equilibrium with that chosen by a constrained social planner.

We assume that the discount rate in the economy is $\beta = .96$, that the risk-free global interest rate satisfies $\beta R = 1$ and that the period utility of domestic agents is CRRA with a coefficient of relative risk aversion $\theta = 2$. We normalize w.l.o.g. $\bar{Y}_T = \bar{Y}_N = 1$. Following Mendoza (2005) we set the relative size of the tradable sector to $\frac{1}{1+\sigma} = .40$ of total output, implying that $\sigma = 1.5$, and we set investment to one third of tradable output, or $\bar{I} = .33$. We use the findings of Reinhart et al. (2003) to calibrate the maximum debt level of a country to 50% of its GDP, implying $\kappa = .50$. We assume furthermore that the output shock $Y^{\omega}_{T,1}$ is distributed according to a truncated normal distribution with standard deviation $\sigma_Y = 0.05$ that is cut off at $\pm 3\sigma_Y$, approximating the standard deviation of growth in a country like Indonesia over the past two decades. This implies that $Y^{\min}_{T,1} = .85$.

5.1 Risk-Neutral International Capital Markets

If international capital markets are risk-neutral, then $M^{\omega}_1 = 1/R$ and the budget constraint of domestic agents implies

$$E[B^{\omega}_1] = R\left(\bar{I} + B_0\right) \tag{29}$$

Furthermore, risk-neutral capital markets enable domestic agents to costlessly insure away all their period 1 consumption risk with international investors.

Loose Constraints If financial constraints are loose across all states of the world, the steady-state level of repayments is $B_{t\geq 2} = \bar{B} := R\left(\bar{I} + B_0\right)$. Both decentralized agents and a social planner in such an economy choose repayments and consumption

$$\left.\begin{array}{ll} B^{\omega}_{t\geq 2} & = E[B^{\omega}_1] = \bar{B} \\ C^{\omega}_{T,t\geq 1} & = \bar{Y}_T - \bar{I} - (R-1)/R \cdot \bar{B} = \bar{C} \end{array}\right\} \quad \text{for all} \quad \omega \in \Omega$$

across all time periods and states. This full insurance result is obtained by contracting state-contingent period 1 repayments that undo the shock,

$$B^{\omega}_1 = \bar{B} + \left(Y^{\omega}_{T,1} - \bar{Y}_{T,1}\right)$$

The condition under which financial constraints are loose in all states is

$$B_0 \leq \hat{B}_0 := \frac{\kappa R}{R + \kappa\sigma(R-1)} \cdot \left[Y^{\min}_{T,1} + \sigma\left(\bar{Y}_T - \bar{I}\right)\right] - \bar{I}$$

Following proposition 3, a social planner for whom this condition is satisfied would make the same financing choices as decentralized markets, i.e. the free market equilibrium is socially efficient.

Binding Constraints If the initial debt level is higher than the threshold $\hat{B}_0$, then financing constraints are binding in low states of period 1. However, domestic agents still find it optimal to fully offload their period 1 endowment risk and consume a constant $C^{\omega}_{T,1} = \bar{C}_{T,1}$ across all states of nature that period. In states of binding constraints, they borrow the maximum amount possible $B^{\omega,con}_2$ and insure period 1 consumption by reducing the repayments B^{ω}_1 in constrained states while increasing repayments in unconstrained states correspondingly. Following proposition 1 for the case of risk-neutral international investors, the equilibrium value of $\bar{C}_{T,1}$, together with the associated financing choices $\{B^{\omega}_1\}$, is obtained as the solution to the implict equation

$$E[B^{\omega}_1(\bar{C}_{T,1})] = R\left(\bar{I} + B_0\right)$$

$$\text{where} \quad B^{\omega}_1(\bar{C}_{T,1}) = Y^{\omega}_{T,1} - \bar{I} - \bar{C}_{T,1} + \left\{ \begin{array}{ll} \kappa\left[Y^{\omega}_{T,1} + \sigma\bar{C}_{T,1}\right] & \text{for} \quad \omega \in \Omega^{con} \\ \frac{1}{R-1}\left[\bar{Y}_T - \bar{I} - \bar{C}_{T,1}\right] & \text{for} \quad \omega \in \Omega^{unc} \end{array}\right.$$

The left panel of figure 3 illustrates the privately optimal financing choices of decentralized agents for differing levels of initial debt. In the figure, we capture the different states of the world by the output shock, which varies from $Y^{\min}_{T,1}$ to $Y^{\max}_{T,1}$ along the horizontal axis. The graph depicts the optimal allocation of period 1 repayments $\{B^{\omega}_1\}$ (solid lines) and long-run debt $\{B^{\omega}_{t\geq 2}\}$ (dashed lines) for increasing levels of initial debt, going from .70 to .75 and .80, as labeled by the roman numerals (i), (ii) and (iii). The period 0 threshold above which there are states with binding constraints is $\hat{B}_0 = .744$ in our sample specification.

Looking at the $B^{(i)}_1$-line, domestic agents choose lower repayments in bad states and higher repayments in good states of nature for insurance purposes so as to obtain a constant level of consumption $C^{\omega}_{T,1} = \bar{C}_{T,1}$ across all states. As long as $B_0 \leq \hat{B}_0$, i.e. for case (i) in the figure, financial constraints never bind, and domestic agents are left with a constant level of steady-state debt $B^{\omega}_{t\geq 2}$ across all states. This results in a constant level of steady state consumption $C_{T,t\geq 2}$ across all states of nature (see the top line (i) in the right panel).

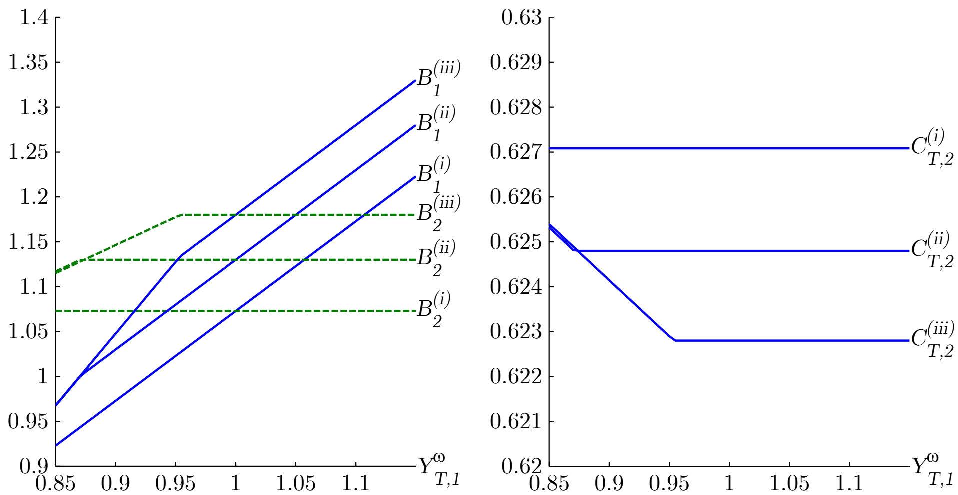

Figure 3: State-contingent repayments (left panel) and consumption (right panel) across different states $Y^{\omega}_{T,1}$ as a function of initial debt B_0

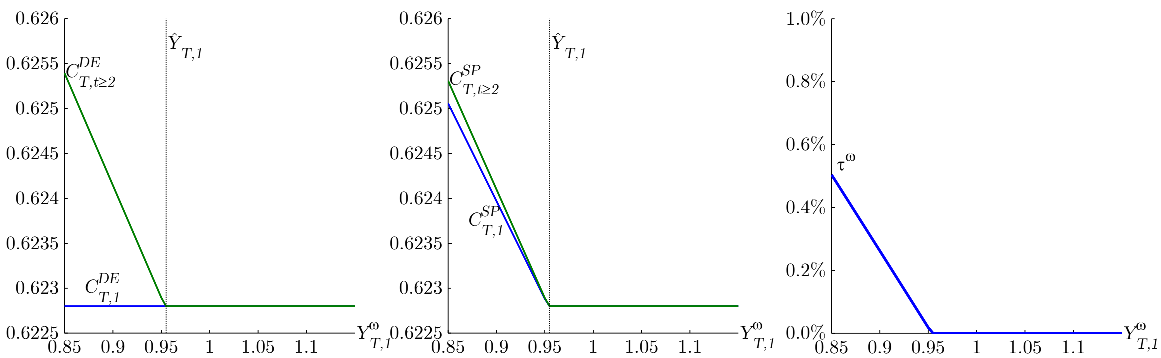

Figure 4: Decentralized and constrained optimal consumption and externality kernel for case of risk-neutral international investors

If initial debt B_0 exceeds the threshold $\hat{B}_0$, there are binding financial constraints in low output states of period 1. Decentralized agents cannot roll over as much debt into period 2 as they would like, i.e. B_2^{ω} is constrained (see the kink in the lines $B_2^{(ii)}$ and $B_2^{(iii)}$). They insure against the inability to borrow the optimum amount by reducing their period 1 repayments when constraints bind (see the kink in lines $B_1^{(ii)}$ and $B_1^{(iii)}$). As a result of the lower long-run debt levels, steady-state consumption $C^{\omega}_{T,t\geq 2}$ is higher following constrained states (right panel).

A constrained social planner recognizes that the tightness of constraints is not a given: higher period 1 tradable consumption acts as a second-best instrument to appreciate the real exchange rate, raise the value of domestic collateral and relax the financial constraint. Therefore she contracts even lower repayments in constrained states of period 1 and enables domestic agents to consume more in such states.

In the left panel of figure 4, we show the consumption allocations of decentralized agents for case (iii) in the previous figure. In the center panel we depict the consumption allocations chosen by a constrained planner. Both decentralized agents and the planner insure away the output shock in unconstrained states, but the constrained planner reduces repayments in constrained states of period 1 further so as to have additional resources available beyond what is required for full insurance: she engages in macroprudential insurance to relax the binding financial constraints and mitigate the financial amplification effects. When international capital markets are risk-neutral, this leads to the curious result that a planner instructs agents to consume more in bad states of nature – so as to relax the constraints – than in good states. We show in the following section that this finding disappears when insurance is sufficiently costly.

The right panel of the figure depicts the externality kernel τ_1^ω of the economy for case (iii), i.e. the externalities imposed by unit payoffs in different states of nature. This is the wedge that a social planner would have to impose on payoffs across different states of nature to induce decentralized agents to internalize their pecuniary externalities.

5.2 Risk-Averse International Capital Markets

When international investors are risk-averse, their pricing kernel M_1^ω covaries negatively with the domestic output shock $Y_{T,1}^\omega$, i.e. they put a relatively higher value on payoffs when the productivity shock $Y_{T,1}^\omega$ is low and vice versa. While our analytical results hold for any general specification of M_0^ω, we assume for simplicity that M_1^ω derives from a consumption-based asset pricing model in which the utility of international investors is CRRA with $\theta = 2$ and their consumption process is given exogenously by $D^\omega = \psi Y_{T,1}^\omega + (1-\psi)\,\bar{Y}_T$, where we calibrate $\psi = .125$ to capture the share of emerging market investments in the portfolio of a representative global investor.[15] As a result, the choice of B_1^ω involves a risk-return trade-off for domestic agents.

Using the same parameter values for domestic agents as in the scenario with risk-free capital markets above, we follow the steps described in proposition 1 to derive the equilibrium. We depict our results in figure 5. Cases (i), (ii) and (iii) depict the allocations in the economy for initial debt levels $B_0 = .25$, .50 and .75 respectively.

In case (i), financial constraints are loose across all states of nature. The optimal insurance arrangement between domestic agents and international investors implies that consumption $C_{T,t\geq 1}^\omega$ is pro-cyclical, reflecting that it is optimal not to insure away all risk in risk-averse international capital markets. Since international investors value payoffs in low states the highest, domestic consumers promise high payoffs in low states and low payoffs in high states. This makes the emerging economy's level of indebtedness counter-cyclical, as captured by the downward-sloping $B_2^{(i)}$-line. In period 1, domestic consumers experience the domestic shock $Y_{T,1}^\omega$; they offset this shock by in-

[15] Even though the economy we examine is small compared to international capital markets, it is reasonable to characterize international investors as risk averse towards the emerging market economy: First, many of the shocks to the tradable sector in emerging market economies are correlated with global factors, such as global financial shocks or fluctuations in commodity prices that are driven by the global business cycle. Secondly, investors into emerging market are a specialized sub-set of actors in global capital markets. Third, if international capital markets were neutral towards emerging market risk, then decentralized agents could insure their economies costlessly against aggregate shocks, which is counter-factual (see Korinek, 2009).

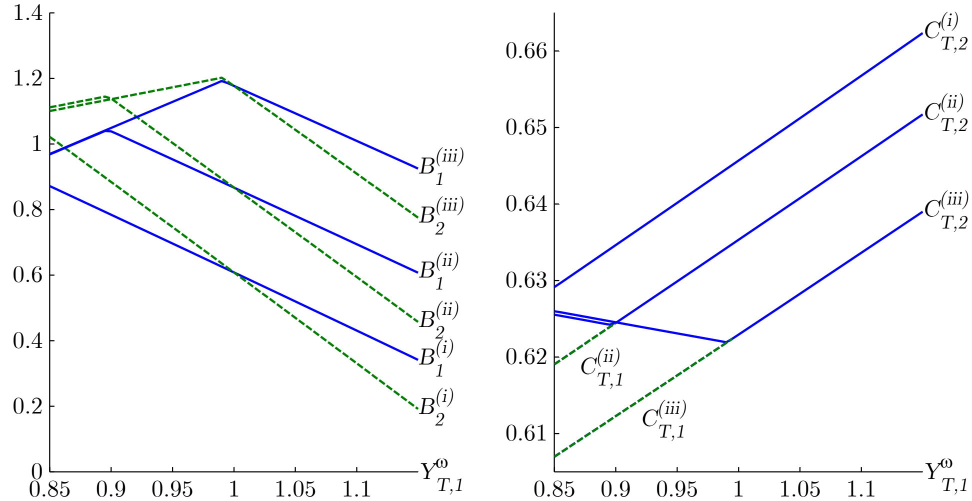

Figure 5: Decentralized and constrained optimal allocation of period 1 consumption across different states $Y^{\omega}_{T,1}$

creasing/decreasing their indebtedness between periods 1 and 2 correspondingly, i.e. by setting $B_1^{\omega} - B_2^{\omega} = Y^{\omega}_{T,1} - \bar{Y}_T$. The $B_1^{(i)}$-line in the figure is therefore less steep than the $B_2^{(i)}$-line.

Cases (ii) and (iii) depict cases in which financing constraints become binding for low realizations of the output shock. Consumption $C^{\omega}_{T,1}$ is still a strictly increasing function of the state of nature, reflecting the optimal insurance arrangement between domestic agents and international investors. However, since constraints are tighter in bad times, the emerging economy's indebtedness becomes pro-cyclical in constrained regions – lower output forces domestic agents to cut back on debt, even though they would like to carry forward more debt so as to optimally smooth consumption. Anticipating the constraint on new debt B_2^{ω} in period 1, domestic agents contract lower contingent period 1 repayments B_1^{ω}, i.e. they take on insurance. Period 2 consumption is a function of period 2 debt levels and is therefore V-shaped. In constrained states, domestic agents cannot carry as much debt into period 2 as they would like; therefore they have less period 2 debt and more consumption in period 2 and thereafter.

The first two panels of figure 6 compare the consumption allocations in the decentralized equilibrium and the constrained social planner's optimum for the initial debt level defined in case (iii): decentralized agents take constraints in period 1 as given and pick a consumption profile that is strictly increasing in the state of the world (left panel). By contrast, a constrained planner engages in additional macro-precautionary savings in period 0; this allows him to raise period 1 consumption in constrained states and relax the period 1 constraint (center panel). As a result, period 1 consumption is considerably less volatile across states of nature under the planner's allocation. Since she repays less in constrained states, the planner has to repay more in previously unconstrained states so as to meet the period 0 budget constraints. This implies that the threshold for binding constraints $\hat{Y}_{T,1}$ moves marginally to the right.[16]

[16]Analytically, we find the thresholds $\hat{\omega}^{DE} = 2.02120$ in the decentralized equilibrium and $\hat{\omega}^{SP} =$

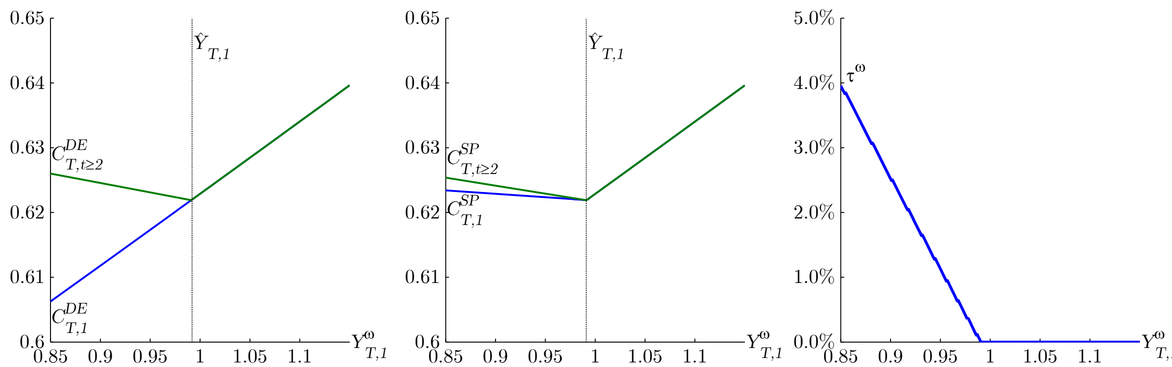

Figure 6: Decentralized and constrained optimal consumption and externality kernel for case of risk-averse international investors

In the right panel, we depict the externality kernel τ_1^ω of economy, i.e. the externality imposed by a unit payoff in different states of nature. In unconstrained states, this is zero. In constrained states, the externality kernel rises in proportion to the wedge in the Euler equation of domestic agents. Comparing figures 4 and 6, we observe that the externality kernel is an order of magnitude higher when capital markets are risk-averse, since domestic agents have greater incentives to issue risky claims that pay off in low states of nature in such a setting.

We derived our results in a model where domestic agents did not fully insure against crisis risk because of the risk premia charged by international investors. While risk premia are an important factor in making full insurance undesirable, there are a number of additional factors, such as costly state verification (Townsend, 1979) or moral hazard (Jensen and Meckling, 1976) that create a private incentive to issue risky claims that expose domestic agents to the risk of binding constraints, or that create a dis-incentive to buy insurance. Our underinsurance result continues to hold in those instances, as domestic agents value only the private benefit of insuring against crises and disregard the social benefits.

In section 7 we employ a sufficient statistics approach to calibrate the size of the externalities of different forms of capital flows – this quantification depends only on the existence of binding constraints, not on the precise mechanism that led individuals to take on risky forms of finance and expose themselves to such constraints. Our calibration results in that section are therefore more general and more robust than the analytical examples we have included here.

5.3 Contagion

In the framework discussed earlier, episodes of binding financial constraints arise from two separate factors: (i) declines in domestic output $Y^\omega_{T,1}$ reduce the economy's financing capacity and may lead to binding constraints even if international capital markets are risk-neutral, and (ii) international risk aversion induces domestic agents to promise large repayments B_1^ω in bad states of nature. If the collateral available $\kappa[Y^\omega_{T,1} + \sigma C^\omega_{T,1}]$

2.02126 in the social optimum. This difference is not sufficient to be visible in the figure.

does not cover the desired amount of debt to be rolled over into period 2, then financial constraints bind. Our example on risk-neutral international capital markets demonstrated that factor (i) may be sufficient to trigger binding constraints, but does not explain declines in consumption in period 1, since domestic agents would insure against such declines in international markets. This section shows that even in the absence of domestic shocks, factor (ii) may trigger crises that are characterized by binding constraints, large current account reversals and declines in consumption. This can be interpreted as contagion through contingent liquidity flows.

We assume for simplicity that output in the emerging market economy is constant, $Y^{\omega}_{T,1} \equiv \bar{Y}_T \; \forall \omega$, but we relax assumption 1: let the pricing kernel of international investors be an arbitrary random variable M^{ω}_1 that satisfies $E[M^{\omega}_1] = \beta$. The equilibrium in this economy can then be solved along the steps outlined in section 3 and appendix A.2. Specifically, lemma 1 still applies for a given state ω, and by implication equation (22) uniquely defines an increasing function $\mu_0(B_0)$. Finally, this enables us to obtain the repayments and consumption allocations B^{ω}_1 and $C^{\omega}_{T,1}$ as well as $B^{\omega}_{t\geq 2}$ and $C^{\omega}_{T,t\geq 2}$ across all states of nature.

The emerging market economy will experience "contagion," i.e. declines in consumption and binding financial constraints, in all states of period 1 in which the international pricing kernel exceeds a threshold $\hat{M}_1$, which is defined as the value for which the constraint marginally binds:

$$B_1(\hat{M}_1, \mu_0(B_0)) = \kappa \left[\bar{Y}_T + \sigma C_{T,1}(\hat{M}_1, \mu_0(B_0)) \right]$$

In all states with $M^{\omega}_1 > \hat{M}_1$, domestic agents repay so much to international investors that their consumption $C^{\omega}_{T,1}$ is depressed, the exchange rate is depreciated, the financial constraint is strictly binding and the economy experiences large capital outflows.

Proposition 6 (Contagion) *In states of high global risk aversion $M^{\omega}_1 \geq \hat{M}^{\omega}_1$, the domestic economy experiences binding constraints. Decentralized domestic agents commit to socially excessive payments to international investors in such states. The resulting equilibrium exhibits socially excessive volatility.*

As in our earlier discussion, decentralized agents do not internalize their contribution to binding financial constraints. By cutting back on consumption, they depreciate the exchange rate and tighten constraints for everybody else. A social planner would save more against such states (macroprudential saving), which allows him to increase consumption as a second-best means of mitigating the constraint and raising welfare.

In practice, we can think of two examples of state-contingent liquidity outflows from emerging market economies that may correspond to our notion of contagion. One results from excessive exposure to risky financial instruments such as credit default swaps, toxic mortgage assets etc. A second state-contingent risk sharing instrument, which has historically been of even greater importance, is short-term debt.[17] If international

[17]While we have not explicitly modeled debts of different maturity structure in our model, short-term debt can be viewed as a contract that pays out very little (interest only) in normal times when such debt is typically rolled over, and that pays out a lot (repayment in full) whenever the liquidity needs of international investors suddenly increase and they call the debt, e.g. in case of global financial turmoil.

capital markets experience a crisis and require liquidity as captured by a high M_1^ω, they simply refuse to roll over short term debts to emerging markets. Whenever the resulting capital outflows are sufficiently large, financial constraints become binding and the externality is triggered.

Emerging market agents provide too much insurance to international investors in this setting, given the underdeveloped nature of their financial system. In an integrated global financial system it would be desirable that global risks are fully shared among all agents in all countries so that the relative marginal valuation of payoffs equate among all market participants, i.e. $M_1^\omega = \frac{\beta\mu_1^\omega}{E[\mu_1^\omega]}$. However, while emerging market economies *seem* to be integrated into global markets in normal times, their access to international financial markets is state-contingent: it is sharply reduced or even lost whenever financing constraints on the economy become binding. Decentralized agents find it privately optimal to participate in global risk-sharing by taking on risky forms of finance, but they fail to internalize that the level of financial integration of their economy during financial crises is endogenous: their private risk-taking decisions affect the tightness of constraints in states of global crisis. Decentralized agents in the described economy share global risks according to their privately optimal tradeoff between risk and return; a social planner takes on global risk according to her tradeoff between risk, return, *and* the endogenous level of financial integration, as captured by the tightness of financial constraints.

6 Policy Implications

The externality in this paper is a by-product of financial amplification effects, i.e. it arises when there are binding financial constraints that lead to feedback effects between consumption, the level of exchange rates, and the tightness of constraints. Hence first-best policy measures would attempt to break this mitigate the constraints so as to avoid this feedback mechanism.

One way of doing so would be to correct the capital market imperfections that underlie the financing constraints. While welfare will unambiguously improve if financial constraints are completely abolished, it is difficult to predict a priori how a mere relaxation of the constraint as captured by an increase in κ would affect welfare, since the relationship between the tightness of constraints and welfare may be non-monotonic (Matsuyama, 2008).

Other measures that could potentially break the amplification effects would be to alleviate binding constraints by providing transfer payments ("bailouts") or by intervening to appreciate the exchange rate. In fact, both measures may be viewed in similar terms in our model, since support of the real exchange rate can be thought of as public purchases of tradable goods that are rebated to domestic agents so as to raise their tradable consumption.[18] While such measures may be optimal ex post, i.e. once financial constraints are binding, we will show in the following subsection that they create severe moral hazard problems. Under certain specific conditions, such government

[18] Recall that in equilibrium, the real exchange rate is determined by the marginal rate of substitution between tradable and non-tradable goods, as captured by equation (8).

interventions will be fully undone by the private sector.

6.1 Bailout Neutrality

Suppose that government commits to making a state-contingent transfer T^ω to decentralized agents. In constrained states, we assume that the transfer $T^\omega > 0$ with the intention of mitigating the binding constraints; in good states of nature, the government imposes lump-sum taxes, i.e. a negative $T^\omega < 0$, so as to make the policy revenue-neutral in expectation, i.e. $E[M_1^\omega T^\omega] = 0$. Then the following result holds:

Proposition 7 (Bailout Neutrality) *Decentralized agents will fully undo any anticipated state-contingent government transfer T^ω with $E[M_1^\omega T^\omega] = 0$.*

The solution to the decentralized agent's problem (18) is an optimal risk/return trade-off. If the government provides an anticipated transfer T^ω in period 1 that is in expectation revenue-neutral, the agent does not experience any wealth effects in period 0 and faces the same first-order conditions. He finds it optimal to sell state-contingent payoffs in the amount T^ω to undo the effects of the transfer, since the decentralized equilibrium with excessive risk-taking constitutes his private optimum. More specifically, if $\{B_1^\omega, B_{t\geq 2}^\omega, C_{T,1}^\omega, C_{T,t\geq 2}^\omega\}$ describes the allocations in an economy without government intervention, then the equilibrium of an economy with anticipated revenue-neutral transfers T^ω is $\{B_1^\omega + T^\omega, B_{t\geq 2}^\omega, C_{T,1}^\omega, C_{T,t\geq 2}^\omega\}$, i.e. aside from the period 1 repayments that undo the transfer all allocations are identical.

A possible example of such behavior occurred in Russia during the financial crisis of 2008, where the government had accumulated large amounts of foreign currency reserves, while banks had accumulated equally large amounts of foreign currency debts over the preceeding years, making the financial sector acutely exposed to the unfolding global financial crisis.

Proposition 7 is closely related to the equivalence result of Barro (1974): According to the traditional version of Ricardian equivalence, private agents undo any reallocations of a given tax burden across time, given that they have access to perfect intertemporal capital markets. In our bailout neutrality proposition, private agents undo a government's reallocations of liquidity across different states of nature, given that they have access to perfect state-contingent markets. In both cases, rational agents recognize that the government's budget constraint is ultimately part of their own budget constraint.

Our proposition is subject to similar limitations as the traditional version of Ricardian equivalence. In particular, the result depends on (i) the assumption that those agents who receive transfers have access to a complete set of Arrow-Debreu markets in order to undo the transfers and (ii) the assumption that in expectation, bailouts do not create net redistributions between different categories of agents.[19]

[19] If there is a set of agents in the economy who do not have access to state-contingent markets (e.g. financially-constrained workers), then even an anticipated transfer to this group has real effects and entails positive economy-wide financial amplification effects by expanding aggregate demand and mitigating the fall in the exchange rate. This has important implications for the design of automatic stabilizers that are effective against financial crises.

Naturally, proposition 7 applies only to *anticipated* transfers. If a government transfer was unanticipated, it would have the desired positive effects. By the same token, if a transfer had been anticipated but is unexpectedly withheld, the crisis will be more severe since private agents have taken on additional risk in anticipation of the transfer.

6.2 Second-best Policy Measures

Given the practical limitations of first-best policy measures that we dicussed in proposition 7, it is desirable to analyze second-best policy measures that restore constrained social efficiency. We focus on two forms of such measures, (i) taxes that aim to discourage the use of socially risk forms of finance ex ante and (ii) quantity restrictions that limit capital outflows ex post in the event of financial crises.[20]

Ex-Ante Taxation The goal of ex-ante taxation of capital inflows is to raise the cost of risky forms of finance so as to reflect the uninternalized part of the expected social cost of repayments by decentralized agents.[21] Section 4 discussed that the magnitude of the externalities imposed by a security X was given by the expected product of the planner's externality kernel and the vector of payoffs X^ω of the security. A social planner who imposes a tax of this magnitude on a security X induces agents to internalize the externality associated with capital inflows by raising the cost of capital on each asset to its socially efficient level.

Proposition 8 (Optimal Pigovian Taxation) *The constrained social optimum can be implemented by imposing a tax*

$$t_X^* = E\left[\tau_1^\omega X_1^\omega\right]$$

on any security X with contingent payoffs X_1^ω and by rebating the revenue in lump-sum fashion.

Equivalent policy measures to explicit taxation include unremunerated reserve requirements (URRs) or banking regulations that reduce the relative attractiveness of risky forms of finance and induce a shift towards a liability structure that is less procyclical.

We also note that many tax systems around the world enable companies and households to deduct interest payments on at least some forms of debt from their taxes. This introduces an important bias into the capital structure of economic agents that leads to excessive debt financing and therefore magnifies the externality that we discussed.

It is sometimes argued that capital account regulations are undesirable because (i) they increase the cost of finance for private firms and (ii) they give rise to evasion (see e.g. Forbes, 2005). Let us discuss each of these arguments in turn. First, raising the

[20]There are a number of Ramsey equivalent policy measures that would induce decentralized agents to internalize the full social value of liquidity in constrained states. We focus on the two chosen measures since they correspond most closely to the capital controls implemented in practice (see e.g. Magud and Reinhart, 2007).

[21]An important benefit of such ex ante measures is that they avoid issues of time consistency, which arise for measures that are to be implemented ex post during financial crises.

private cost of capital inflows to the social cost is precisely the point of such regulations (just as environmental regulations raise the cost of pollution in order to discourage it). Secondly, all regulation that imposes costly constraints gives rise to attempts to circumvent it – this includes public health regulations or, to put it starkly, criminal laws against murder. As this latter example illustrates, the existence of attempts to circumvent regulation are not a good reason to abolish it; rather, they should encourage regulators to come up with more robust rules and better ways of enforcement.[22]

Suspension of Capital Flows When financial constraints are binding and a country experiences severe capital outflows (i.e. in states when $B_1^\omega \gg B_2^\omega/R$), a policy measure of last resort has been to temporarily suspend international capital flows.[23] In a rational expectations framework, a policy rule that limits outflows in constrained states of nature would reduce the amount of repayments that domestic agents commit to in such states.

In our analytical framework, such a rule can be captured as a ceiling on repayments $\{\bar{B}_1^\omega\}$ across the different states of nature.

Proposition 9 (Contingent Suspension of Capital Flows) *The constrained social optimum can be implemented by imposing a state-contingent ceiling $\bar{B}_1^\omega = B_1^{\omega,SP}$ on repayments in constrained states of nature and by setting $\bar{B}_1^\omega = \infty$ in unconstrained states.*

By implication, a credible and well-defined framework of conditional suspension of capital flows, verified by an independent international arbiter such as the IMF, would improve welfare by limiting risk-taking ex ante. In other words, given the full set of Arrow securities in our framework, the existence of rules that limit repayments in certain states of nature would bias the contingent liability structure of domestic agents towards less risk-taking. This would make domestic agents better off while leaving the welfare of international investors unaffected.

A policy rule to suspend capital flows during severe crises would also solve the dilemma created by the bailout neutrality results in proposition 7, as decentralized agents can no longer undo the effects of anticipated bailouts during financial crises: even if they attempted to take on additional risk, international investors would know that the promised payments could not be repatriated when capital flows are suspended.

Financial Stability and Allocative Efficiency It is sometimes argued that policymakers that decide on the optimal level of financial regulation face a Pareto-frontier along which there is a trade-off between financial stability vs. allocative efficiency, akin to the tradeoff of risk vs. return faced by portfolio investors. According to this view, any form of financial regulation enhances stability at the expense of efficiency. Our paper

[22]One proposal to enforce regulations on capital inflows is to make claims by foreign creditors that have evaded capital controls unenforceable in court.

[23]In practice, ex-post quantity controls are considerably more difficult to implement than ex-ante taxes, because they require the ex post verification of the state of nature. However, they have been commonly used as a measure of last resort: for example, Radelet and Sachs (1998) describe that the downward spiral and financial meltdown in the countries of the East Asian crisis came to an end only when a temporary suspension of debt payments and a forced roll-over of debts was implemented, brokered by the IMF and the US government.

clearly rejects this view: since there exist externalities in economies that are prone to financial amplification effects, financial regulation can simultaneously enhance stability *and* efficiency. In other words, in the absence of regulation, an economy is inside the Pareto frontier, and well-designed regulation can make everybody better off.

7 A Sample Calibration

In order to investigate the quantitative relevance of our results empirically, we perform a sample calibration of an externality kernel and derive the optimal tax on a number of different forms of capital flows. We base our sample calibration on yearly historical data of Indonesia over the past two decades (1989 – 2008).[24]

Our calibration method is in the spirit of the sufficient statistics approach of Chetty (2009): we show that the externality kernel of a country that is prone to amplification effects can be expressed as the product of two sufficient statistics that impose only minimal requirements on data availability: the wedge in the Euler equation of decentralized agents and the extent of amplification effects.

There are two important benefits to this approach: First, it is independent of the specific model structure that we assumed in earlier sections. Our calibration is valid for *any* model of financial amplification effects in which a country's external financing capacity and the tightness of its constraints depend endogenously on the level of the exchange rate (see e.g. Krugman, 1999; Schneider and Tornell, 2004; Mendoza, 2005, among many others). In particular, the results of our calibration are robust to the simplifying assumptions we made in our analytical model, such as uncertainty in period one only, or the simple model of the exchange rate we employed, and the precise specification of financial constraints. Secondly, it does not require estimating the structural parameters of the model, some of which might be difficult to obtain empirically and not robust to small changes in the economic environment, such as the parameters governing the financial constraint. In that sense, our calibration approach and the magnitude of the externalities that we obtain are more robust and reliable than what can be obtained by standard calibration exercises that rely on a specific model structure. Our approach therefore provides for a more direct, transparent and credible identification of the described externalities.

Let us note three caveats of our estimation approach below (which are, however, unrelated to the sufficient statistics approach taken): First, since our calibration is based on historical data, it does not account for permanent changes in the structure of the economy that might occur over time. Secondly, we assume the externality kernel is constant over time. Thirdly, we implicitly assume a yearly maturity for the assets we investigate. Financial payoffs that are of shorter (longer) maturity than one year may impose larger (smaller) externalities. The framework could be adjusted to take these factors into account, but this is beyond the scope of the illustration contained in the current paper.

Our proposed sufficient statistics approach consists of five steps:

[24]Data from International Financial Statistics, IMF, 2009.

Step 1: *Describe the set Ω of potential outcomes and identify constrained states:* In our example, we define $\Omega = \{1989, \ldots, 2008\}$ as the state space, which we assume representative for the Indonesian economy, i.e. we assign to each of these states a probability $\pi_\omega = 5\%$. During the described time span, the Asian financial crises of 1997/98 is the only incident in which a currency crises as defined by Frankel and Rose (1996) and a sudden stop as defined by Calvo (1998) took place. The crisis in Indonesia started in fall of 1997 and culminated in 1998 (Radelet and Sachs, 1998). Since we are using yearly data, we attribute the entire crisis to the calendar year of 1998.

Step 2: *Quantify the tightness $\frac{\lambda^\omega_{DE}}{E[\mu^\omega_{DE}]}$ of constraints:* The tightness of constraints (as perceived by decentralized agents) is given by the wedge in their Euler equation, which we normalize by the expected marginal utility of decentralized agents. Real consumption data during the crisis is unreliable due to the difficulties of estimating proper deflators in a high inflation environment, hence we approximate the wedge by using the economy's percentage decline in real GDP $\Delta y^{1998} = -13.1\%$ as a guide for the decline in consumption and the difference in relative marginal utilities that was experienced. Assuming a CRRA utility function with coefficient of relative risk aversion $\gamma = 2$, a Taylor-approximation of this wedge yields

$$\frac{\lambda^{1998}_{DE}}{E[\mu^\omega_{DE}]} \approx -\gamma \cdot \Delta y^{1998} = 26.2\%$$

We assumed in step 1 that Indonesia experienced no other episode of economy-wide binding financial constraints, so we set $\lambda^\omega_{DE} = 0$ for all other states of the world $\omega \neq 1998$.

Step 3: *Estimate the strength of amplification effects:* The factor $\frac{\kappa\sigma}{1-\kappa\sigma}$ in equation (28) captures how strongly a given change in aggregate demand affects the tightness of financial constraints K^ω in our model at the margin, i.e. $\frac{\kappa\sigma}{1-\kappa\sigma} = \frac{dK^\omega}{dY^\omega}$. For data availability reasons, we approximate this marginal effect using the average change in financing capacity. We capture this by the magnitude of the observed current account reversal, $\Delta CA/Y$, expressed in percent of GDP, that results from a change in aggregate demand $\Delta Y/Y$ in constrained states:

$$\frac{\kappa\sigma}{1-\kappa\sigma} = \frac{dK^{1998}}{dY^{1998}} \approx \frac{(\Delta CA/Y)^{1998}}{(\Delta Y/Y)^{1998}} = \frac{-7.1\%}{-13.1\%} = .54$$

We express the externality kernel as[25]

$$\tau^{1998} = \frac{dK^{1998}}{dY^{1998}} \cdot \lambda^{1998}_{DE} = .54 \cdot 26.2\% = 14.1\%$$

In other words, we estimate the externalities caused by a capital outflow from Indonesia in 1998 to be equivalent to 14.1% of the amount of the outflow.

[25] In a more general model, the fraction $\frac{\kappa\sigma}{1-\kappa\sigma}$ might vary across different episodes of binding constraints and could be calibrated state-by-state as $\left(\frac{\kappa\sigma}{1-\kappa\sigma}\right)^\omega$. In the given example, this issue is not relevant since we identified only one episode of binding constraints for Indonesia.

Asset category	Real gross return	Externality in 1998	Optimal tax
Dollar debt	218%	30.7%	1.54%
GDP-indexed dollar debt	190%	26.8%	1.34%
CPI-indexed rupiah debt	100%	14.1%	0.71%
Rupiah debt	63%	8.9%	0.44%
Stock market index	44%	6.2%	0.31%

Table 1: Realized gross excess returns and externalities of different asset categories in Indonesia, 1998.

Step 4: *Describe the payoff structure X^ω of different assets in constrained states:* In order to obtain the externalities caused by different forms of capital flows, we need to characterize the state-contingent payoffs of different asset classes in constrained states. We compile a list of the realized gross excess returns of different asset categories in 1998 measured in real domestic consumption units (i.e. local currency units deflated by consumer prices) in the first column of table 1.

Step 5: *Calculate the expected magnitude of the externality as $E[\tau^\omega X^\omega]$:* The externality created by each payoff in a given state ω can simply be obtained as the realized real gross return X^ω multiplied by the externality kernel τ^ω. We have calculated this for Indonesia in 1998 in the second column of table 1.

Finally, since we assumed that the set Ω is representative of the long-run incidence of adverse shocks and binding constraints, we express the expected magnitude of the externality by multiplying this number with the probability of the state $\pi^{1998} = 5\%$. The results are given in the final column of the table.

We find the following pecking order of capital flows, ranked by the externalities they impose on the recipient country:

- **Dollar debt** is characterized by large repayments in constrained states of the world and therefore imposes a considerable externality on the borrowing economy. Calibrated to the case of Indonesia, we found the size of this externality to be 30.7% in 1998, or an unconditional 1.54% per year in the long run decentralized equilibrium.

- **GDP-indexed dollar debt** is dollar-denominated debt with coupon payments that are linked to the country's rate of growth. In Indonesia in 1998, the return on such GDP-linked debt would have accordingly been by 13.1% less than on non-indexed dollar debt. Since Indonesia's exchange rate depreciated so sharply in 1998, the gross return on such debt would have been 190%, imposing an externality of 26.8% on the economy. GDP-linked debt that is denominated in foreign currency is therefore not a good insurance instrument for emerging market economies.

- **CPI-indexed rupiah debt** acts like a real bond that is denominated in domestic units of consumption. As such they protect investors from the inflation that typically occurs during crises, but they do not expose borrowers to the pro-cyclicality exhibited by foreign currency debt instruments. In 1998, such debt would have imposed a 14.1% externality on Indonesia.

- **Rupiah debt** lost value as the country's price level was inflated in 1998, which is typical during emerging market crisis. This implies that non-indexed local currency debt is an excellent insurance instrument, exhibiting only an 8.9% externality during the Indonesian crisis of 1998.

- **Portfolio investment** in emerging equity markets looses value in crises, both because share prices denoted in domestic currency fall and the currency itself depreciates. In our sample calibration, an investment in the stock market index created only a 6.2% externality in 1998 – one fifth of the externality imposed by an inflow of dollar debt of identical magnitude.

- **Foreign direct investment** (omitted from table) is unlikely to yield profits during crises and therefore to entail profit repatriations and capital outflows. From this point of view, foreign direct investment is the one form of finance that does not create negative externalities.[26]

8 Conclusions

This paper showed that the financial amplification effects that arise during emerging market financial crises create an externality that induces decentralized agents to take on excessively risky forms of finance and expose the economy to too much systemic risk. The resulting macroeconomic equilibrium exhibits socially excessive volatility in the tightness of financial constraints, in capital flows, and in consumption.

We described the necessary building blocks of an optimal regulatory system for international capital flows in emerging market economies that makes decentralized agents internalize the externalities they impose on the rest of the economy. This would allow emerging market economies to enjoy the benefits of financial globalization while mitigating the pitfalls in the form of frequent financial crises, thereby increasing social welfare.

Aside from distorting financing decisions in an emerging market economy, we expect that the same externality also creates distortions in their investment decisions: First, since decentralized agents do not internalize the full social cost of capital, they would generally invest too much (see e.g. Lorenzoni, 2008, for a closed economy example). Secondly, since they undervalue payoffs in constrained states of nature, their investment decisions would be biased towards excessively pro-cyclical projects and they would undervalue the benefits of diversification. These questions are the subject of our ongoing research.

[26] If a parent company injects additional liquidity into its emerging market subsidiary during crises, then the resulting capital inflows entail positive externalities that would call for a subsidy, since they raise aggregate demand and mitigate financial amplification.

While the current paper has analyzed this externality in an open economy model of amplification effects, a similar mechanism arises more generally whenever an economy is subject to financial amplification effects whereby declines in certain prices (such as exchange rates, asset prices etc.) and reductions in output (e.g. because of balance sheet effects) mutually reinforce each other. The essential feature is that atomistic agents take macroeconomic prices as given, even though price declines exacerbate the tightness of constraints that play such an important role in financial amplification effects. For a related analysis in a closed-economy model where financial crises entail feedback effects between declining asset prices and tightening financial constraints see Korinek (2010).

References

Aghion, P., Bacchetta, P., and Banerjee, A. (2004). A corporate balance-sheet approach to currency crises. *Journal of Economic Theory*, 119(1):6–30.

Barro, R. J. (1974). Are government bonds net wealth? *Journal of Political Economy*, 82(6):1095–1117.

Benigno, G., Chen, H., Otrok, C., Rebucci, A., and Young, E. R. (2013). Capital controls or exchange rate policy? a pecuniary externality perspective. *mimeo*.

Bernanke, B., Gertler, M., and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. In Taylor, J. B. and Woodford, M., editors, *Handbook of Macroeconomics*, volume 1C, pages 1341–1393. Elsevier.

Bhagwati, J. (1998). The capital myth: The difference between trade in widgets and dollars. *Foreign Affairs*, 7(3):7–12.

Bianchi, J. (2011). Overborrowing and systemic externalities in the business cycle. *American Economic Review*, 101(7):3400–3426.

Caballero, R. J. and Krishnamurthy, A. (2003). Excessive dollar debt: Financial development and underinsurance. *Journal of Finance*, 58(2):867–894.

Calvo, G. A. (1998). Capital flows and capital-market crises: The simple economics of sudden stops. *Journal of Applied Economics*, 1(1):35–54.

Calvo, G. A., Izquierdo, A., and Mejía, L.-F. (2004). On the empirics of sudden stops - the relevance of balance-sheet effects. *NBER Working Paper*, w10520.

Chetty, R. (2009). Sufficient statistics for welfare analysis: A bridge between structural and reduced-form methods. *Annual Review of Economics*, 1:451–487.

Christiano, L. J., Gust, C., and Roldos, J. (2004). Monetary policy in a financial crisis. *Journal of Economic Theory*, 119(1):64–103.

Costinot, A., Lorenzoni, G., and Werning, I. (2011). A theory of capital controls as dynamic terms-of-trade manipulation. *NBER Working Paper*, w17680.

Devereux, M. B. and Yetman, J. (2009). Financial deleveraging and the international transmission of shocks. *BIS Working Paper*.

Eichengreen, B. and Hausmann, R. (1999). Exchange rates and financial fragility. In *New Challenges for Monetary Policy*, pages 329–368. Federal Reserve Bank of Kansas City, Kansas City, Missouri.

Farhi, E. and Werning, I. (2012). Dealing with the trilemma: Optimal capital controls with fixed exchange rates. *Harvard University, mimeo.*

Farhi, E. and Werning, I. (2013). Dilemma not trilemma? capital controls and exchange rates with volatile capital flows. *Harvard University, mimeo.*

Fisher, I. (1933). The debt-deflation theory of great depressions. *Econometrica*, 1(4):337–357.

Forbes, K. J. (2005). The microeconomic evidence on capital controls: No free lunch. *NBER Working Paper 11372*.

Frankel, J. A. and Rose, A. K. (1996). Currency crashes in emerging markets: An empirical treatment. *Journal of International Economics*, 41(3):351 – 366.

Gertler, M. and Kiyotaki, N. (2010). Financial intermediation and credit policy in business cycle analysis. In *Handbook of Monetary Economics*, volume 3, pages 547–599. Elsevier.

Gourinchas, P.-O. and Jeanne, O. (2006). The elusive gains from international financial integration. *Review of Economic Studies*, 73:715–741.

Jeanne, O. (2003). Why do emerging economies borrow in foreign currency? *IMF Working Paper*, 03/177.

Jeanne, O. (2009). Debt maturity and the international financial architecture. *American Economic Review*, 99(5):2135–2148.

Jeanne, O. and Zettelmayer, J. (2005). "original sin," balance sheet crises and the roles of international lending. In Eichengreen, B. and Hausmann, R., editors, *Other People's Money: Debt Denomination and Financial Instability in Emerging Market Economies*. University of Chicago Press, Chicago.

Jensen, M. C. and Meckling, W. H. (1976). Theory of the firm: Managerial behavior, agency costs, and ownership structure. *Journal of Financial Economics*, 3:305–360.

Kiyotaki, N. and Moore, J. (1997). Credit cycles. *Journal of Political Economy*, 105(2):211–248.

Korinek, A. (2009). Spanning properties of dollar and local currency debt. *University of Maryland, mimeo.*

Korinek, A. (2010). Systemic risk-taking: Amplification effects, externalities, and regulatory responses. *University of Maryland, mimeo.*

Krugman, P. R. (1998). What happened to asia?

Krugman, P. R. (1999). Balance sheets, the transfer problem, and financial crises. In Isard, P., Razin, A., and Rose, A. K., editors, *International Finance and Financial Crises: Essays in Honor of Robert P. Flood Jr.*, pages 31–44. International Monetary Fund, Washington, DC.

Levy Yeyati, E. (2006). Financial dollarization: evaluating the consequences. *Economic Policy*, 21(45):61–118.

Lorenzoni, G. (2008). Inefficient credit booms. *Review of Economic Studies*, 75(3):809 – 833.

Magud, N. and Reinhart, C. M. (2007). Capital controls: An evaluation. In Edwards, S., editor, *Capital Controls and Capital Flows in Emerging Economies: Policies, Practices and Consequences*, pages 645–674. University of Chicago Press.

Matsuyama, K. (2008). Aggregate implications of credit market imperfections. In Acemoglu, D., Rogoff, K. S., and Woodford, M., editors, *NBER Macroeconomics Annual 2007*, pages 1–60. Chicago University Press.

Mauro, P., Ostry, J. D., Dell'Ariccia, G., di Giovanni, J., Faria, A., Kose, A., Schindler, M., and Terrones, M. E. (2007). Reaping the benefits of financial globalization. *IMF Discussion Paper.*

Mendoza, E. G. (2005). Real exchange rate volatility and the price of nontradables in sudden-stop-prone economies. *Economia*, 6(1):103–148.

Obstfeld, M. and Rogoff, K. (1996). *Foundations of International Macroeconomics.* MIT Press.

Radelet, S. and Sachs, J. D. (1998). The east asian financial crisis: Diagnosis, remedies, prospects. *The East Asian Financial Crisis; Diagnosis, Remedies, Prospects, Brookings Papers on Economic Activity*, 1998(1):1–90.

Rancière, R. (2009). Financial liberalization, boom-bust cycles and production efficiency. *mimeo.*

Reinhart, C. M. and Reinhart, V. R. (2008). Capital flow bonanzas: An encompassing view of the past and present. *NBER Working Paper*, w14321.

Reinhart, C. M., Rogoff, K., and Savastano, M. A. (2003). Debt intolerance. *NBER Working Paper*, w9908.

Rodrik, D. (1998). Who needs capital-account convertibility? In Fischer, S., editor, *Should the IMF Pursue Capital-Account Convertibility.* International Finance Section, Department of Economics, Princeton University, Princeton.

Schneider, M. and Tornell, A. (2004). Balance sheet effects, bailout guarantees and financial crises. *Review of Economic Studies*, 71(3):883–913.

Stiglitz, J. E. (2002). *Globalization and its Discontents*. W.W. Norton, New York and London.

Tirole, J. (2003). Inefficient foreign borrowing: A dual- and common-agency perspective. *American Economic Review*, 93(5):1678–1702.

Tobin, J. (1978). A proposal for international monetary reform. *Eastern Economic Journal*, 4(3-4):153–159.

Townsend, R. M. (1979). Optimal contracts and competitive markets with costly state verification. *Journal of Economic Theory*, 21(2):265–293.

A Appendix [online only]

A.1 Optimization Problem of Decentralized Agents

We formulate the Lagrangian associated with the optimization problem in section 2 as

$$\begin{aligned}\mathcal{L} =&E\sum_{t=1}^{\infty}\beta^t \left\{u\left(C^\omega_{T,t}(C^\omega_{N,t})^\sigma\right) - \mu_0\left[B_0 + \bar{I} - E(M^\omega_1 B^\omega_1)\right]\right. \\ &-\mu^\omega_t\left[B^\omega_t + \bar{I} + C^\omega_{T,t} + p^\omega_{N,t}C^\omega_{N,t} - Y^\omega_{T,t} - p^\omega_{N,t}\bar{Y}_N - E(M^\omega_{t+1}B^\omega_{t+1})\right] \\ &\left.-\lambda^\omega_t\left[E(M^\omega_{t+1}B^\omega_{t+1}) - \kappa\big(Y^\omega_{T,t} + p^\omega_{N,t}Y^\omega_{N,t}\big)\right]\right\}\end{aligned} \tag{A.1}$$

This Lagrangian is optimized with respect to the variables $\{C^\omega_{T,t}, C^\omega_{N,t}, B^\omega_{t+1}\}$.

A.2 Decentralized Period 1 Repayment Function

If the financial constraint is loose in state ω of period 1, then consumption will be constant from period 1 onwards. The steady-state level of debt consistent with a level of consumption $C_{T,1}(M^\omega_1, \mu_0)$ is

$$B_{t\geq 2}(M^\omega_1, \mu_0) = \frac{R}{R-1}\left[\bar{Y}_T - \bar{I} - C_{T,1}(M^\omega_1, \mu_0)\right]$$

This requires an optimal period 1 repayment that insures against the productivity shock in period 1,

$$B^{unc}_1(M^\omega_1, \mu_0) = B_{t\geq 2}(M^\omega_1, \mu_0) + Y^\omega_{T,1} - \bar{Y}_T \tag{A.2}$$

which satisfies $\partial B^{unc}_1/\partial\mu_0 > 0$. The resulting steady-state level of debt will indeed satisfy the financial constraint if

$$B_{t\geq 2}(M^\omega_1, \mu_0) \leq \kappa\left[Y^\omega_{T,1} + \sigma C_{T,1}(M^\omega_1, \mu_0)\right] \tag{A.3}$$

If the constraint is binding in some states of nature, then there is a unique threshold $\hat{\omega}(\mu_0) \in \Omega$ that partitions the set Ω into a constrained region $\Omega^{unc}(\mu_0) = \{\omega \in \Omega : \omega < \hat{\omega}(\mu_0)\}$ and an unconstrained region $\Omega^{unc}(\mu_0) = \Omega\backslash\Omega^{con}(\mu_0)$ for a given μ_0. This follows since the left-hand side of the constraint is decreasing in ω whereas the right-hand side is increasing in ω.

In the constrained region, the contingent period 1 repayment is determined by substituting the optimal level of consumption (21) and the binding constraint into the period 1 budget constraint,

$$B^{con}_1(M^\omega_1, \mu_0) = (1+\kappa)Y^\omega_{T,1} - \bar{I} - (1-\kappa\sigma)C_{T,1}(M^\omega_1, \mu_0) \tag{A.4}$$

At the threshold $\hat{\omega}(\mu_0)$, we find that $B_1^{con}(M_1^{\hat{\omega}}, \mu_0) = B_1^{unc}(M_1^{\hat{\omega}}, \mu_0)$. For a given μ_0, the function $B_1(M_1^{\omega}, \mu_0)$ is therefore continuous in M_1^{ω} and satisfies 1.

Equation (22) can be solved uniquely for a function $\mu_0(B_0)$ that is strictly increasing $\partial\mu_0/\partial B_0 > 0$.[27]

The threshold for initial debt $\hat{B}_0$ below which the financing constraint is always loose is defined implicitly by the value of B_0 such that the constraint is marginally binding in the lowest endowment state $\omega_{\min}$ of period 1,

$$B_1^{\text{unc}}(M_1^{\omega_{\min}}, \mu_0(\hat{B}_0)) = \kappa \left[Y_{T,1}^{\omega_{\min}} + \sigma C_{T,1}(M_1^{\omega_{\min}}, \mu_0(\hat{B}_0))\right] \tag{A.5}$$

$\hat{B}_0$ is uniquely defined since the left-hand side of this equation is increasing in B_0 and the right-hand side is decreasing in B_0.

A.3 Threshold for Binding Financial Constraints

Starting from the inequality

$$B_2/R \le \kappa \left(Y_{T,1} + \sigma C_{T,1}\right)$$

we substitute for the unconstrained values $B_2 = B_1 - (Y_{T,1} - \bar{Y}_T)$ and $C_{T,1} = \bar{Y}_T - \bar{I} - \frac{R-1}{R} B_2$ to obtain

$$B_1 + \bar{Y}_T - Y_{T,1} \le \kappa R \left\{ Y_{T,1} + \sigma \left[\bar{Y}_T - \bar{I} - \frac{R-1}{R} \left(B_1 + \bar{Y}_T - Y_{T,1}\right) \right] \right\}$$

Collecting terms yields

$$B_1 \le \hat{B}(Y_{T,1}) := \frac{\kappa R Y_{T,1} + \kappa\sigma R(\bar{Y}_T - \bar{I})}{1 + \kappa\sigma(R-1)} + Y_{T,1} - \bar{Y}_T \tag{A.6}$$

which satisfies $\partial\hat{B}/\partial Y_{T,1} > 0$.

A.4 Thresholds for Initial Debt Level

Decentralized Equilibrium Denote the state of nature for which the economy's endowment takes the average value $Y_{T,1}^{\omega} = \bar{Y}_T$ as $\bar{\omega}$. We define the threshold $\bar{B}_0^{DE}$ as the initial debt level which implies that the constraint is marginally binding in state $\bar{\omega}$ of the decentralized equilibrium:

$$B_1^{unc}(M_1^{\bar{\omega}}, \mu_0(\bar{B}_0^{DE})) = \kappa \left[\bar{Y}_T + \sigma C_{T,1}^{unc}(M_1^{\bar{\omega}}, \mu_0(\bar{B}_0^{DE}))\right] \tag{A.7}$$

This threshold is unique as the left-hand side is increasing in B_0 and the right-hand side is decreasing in B_0.

Constrained Social Optimum We define the threshold $\bar{B}_0^{SP}$ for which the social planner's allocation implies marginally binding constraints in state $\bar{\omega}$ by the implicit equation:

$$B_1^{unc}(M_1^{\bar{\omega}}, \mu_0^{SP}(\bar{B}_0^{SP})) = \kappa \left[\bar{Y}_T + \sigma C_{T,1}(M_1^{\bar{\omega}}, \mu_0^{SP}(\bar{B}_0^{SP}))\right]$$

Since $\mu_0^{SP}(B_0) > \mu_0^{DE}(B_0)$ whenever there are binding constraints in some states of nature, it follows that $\bar{B}_0^{SP} \le \bar{B}_0^{DE}$ with strict inequality if constraints bind in some states of nature.

Therefore we define the threshold relevant for assumption 3 as $\bar{B}_0 = \bar{B}_0^{SP}$.

[27] In applying the Leibniz rule to the expectation term in (22), the terms containing the derivatives of the limit $\partial\hat{\omega}/\partial\mu_0$ drop out because the function $B_1(M_1^{\omega}, \mu_0)$ is continuous at $\hat{\omega}$, i.e. $B_1^{\text{con}}(M_1^{\hat{\omega}}, \mu_0) = B_1^{\text{unc}}(M_1^{\hat{\omega}}, \mu_0)$.

A.5 Optimization Problem of Constrained Planner

The constrained planner solves a problem similar to (A.1), with the only difference that she recognizes that the exchange rate in the constraint (4) is in equilibrium determined by the level of tradable consumption $p^{\omega}_{N,t} = \sigma C^{\omega}_{T,1}$. This modifies the earlier Lagrangian to

$$
\begin{aligned}
\mathcal{L} =& E\sum_{t=1}^{\infty}\beta^t \left\{u\left(C^{\omega}_{T,t}(C^{\omega}_{N,t})^{\sigma}\right) - \mu_0\left[B_0 + \bar{I} - E(M^{\omega}_1 B^{\omega}_1)\right]\right.\\
& - \mu^{\omega}_t\left[B^{\omega}_t + \bar{I} + C^{\omega}_{T,t} + p^{\omega}_{N,t}C^{\omega}_{N,t} - Y^{\omega}_{T,t} - p^{\omega}_{N,t}\bar{Y}_N - E(M^{\omega}_{t+1}B^{\omega}_{t+1})\right]\\
& \left. -\lambda^{\omega}_t\left[E(M^{\omega}_{t+1}B^{\omega}_{t+1}) - \kappa\big(Y^{\omega}_{T,t} + p^{\omega}_{N,t}Y^{\omega}_{N,t}\big)\right]\right\}
\end{aligned}
$$

This Lagrangian is optimized with respect to the variables $\{C^{\omega}_{T,t}, B^{\omega}_{t+1}\}$.

A.6 Planner's Period 1 Repayment Function

By substituting $C^{\omega}_{T,2} = \bar{Y}_T - \bar{I} - \kappa(R-1)[Y^{\omega}_{T,1} + \sigma C^{con}_{T,1}]$ into equation (26), we obtain an expression that implicitly defines the constrained planner's optimal period 1 consumption function $C^{con,SP}_{T,1}(M^{\omega}_1, \mu_0)$ for constrained states $\omega \in \Omega^{con}$:

$$
\frac{u'(C^{con,SP}_{T,1}) - \kappa\sigma u'\left(\bar{Y}_T - \bar{I} - \kappa(R-1)\left[Y^{\omega}_{T,1} + \sigma C^{con,SP}_{T,1}\right]\right)}{1-\kappa\sigma} = \mu_0 M^{\omega}_1 \tag{A.8}
$$

The left-hand side of this expression is strictly decreasing in $C_{T,1}$; therefore the resulting consumption function $C^{con,SP}_{T,1}(M^{\omega}_1, \mu_0)$ is uniquely determined and satisfies $\partial C^{con,SP}_{T,1}/\partial M^{\omega}_1 < 0$ and $\partial C^{con,SP}_{T,1}(\cdot)/\partial\mu_0 < 0$. This level of consumption is obtained by contracting a level of period 1 repayments

$$
B^{con,SP}_1(M^{\omega}_1, \mu_0) = (1+\kappa)Y^{\omega}_{T,1} - \bar{I} - (1-\kappa\sigma)C^{con,SP}_{T,1}(M^{\omega}_1, \mu_0) \tag{A.9}
$$

which is increasing in both M^{ω}_1 and μ_0. (Note that the partial derivatives of the decentralized solutions (21) and (A.4) have the same signs as those of the planner.)